RUB

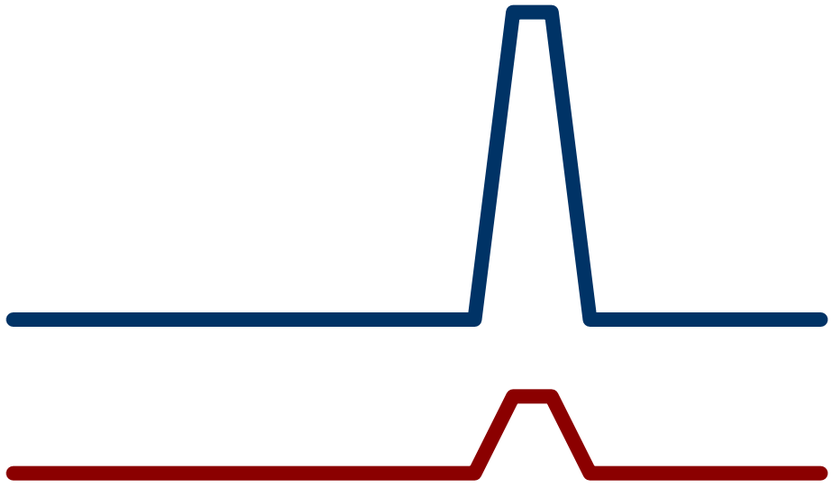
A natural metric for curves – 35 years of Fréchet distance computation

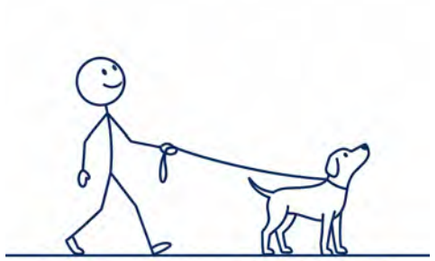
Maike Buchin

EuroCG 2026, Hagen

RUB

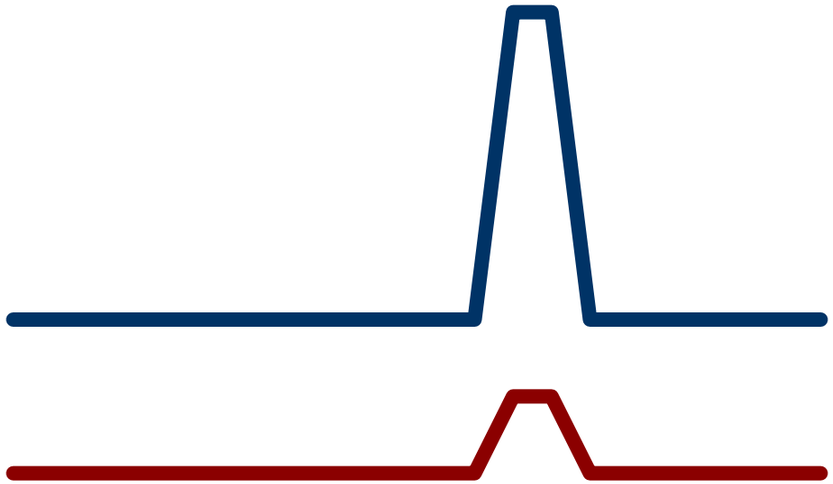
Question: How similar are two polygonal curves?

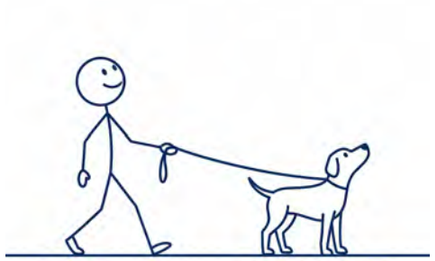


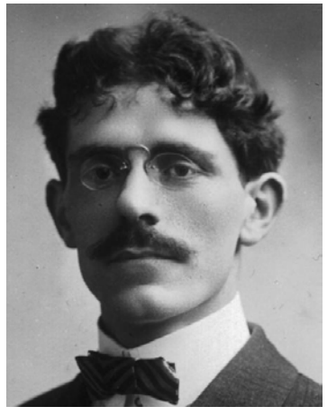


Introduction

An example of a metric for parameterized curves







Maurice Fréchet, 1906

https://perso.lpsm.paris/ mazliak/ConfFrechetang.html

SUR QUELQUES POINTS DU CALCUL FONCTIONNEL;

Par M. **Maurice Fréchet** (Paris) *).

Adunanza del 22 aprile 1906.

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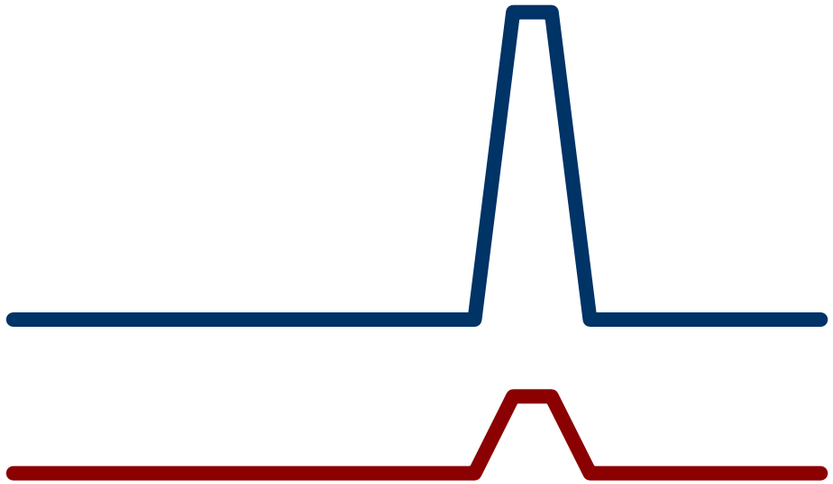
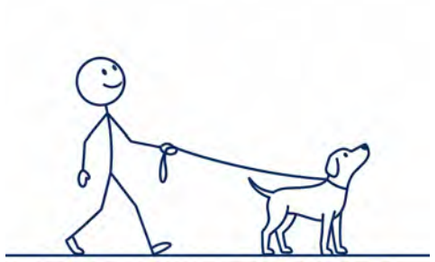
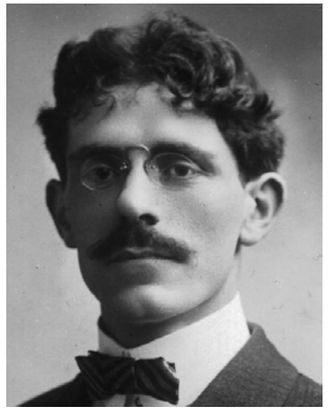
homeomorphism







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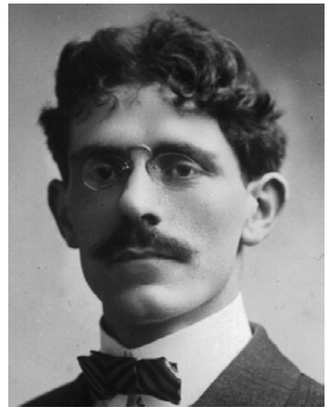

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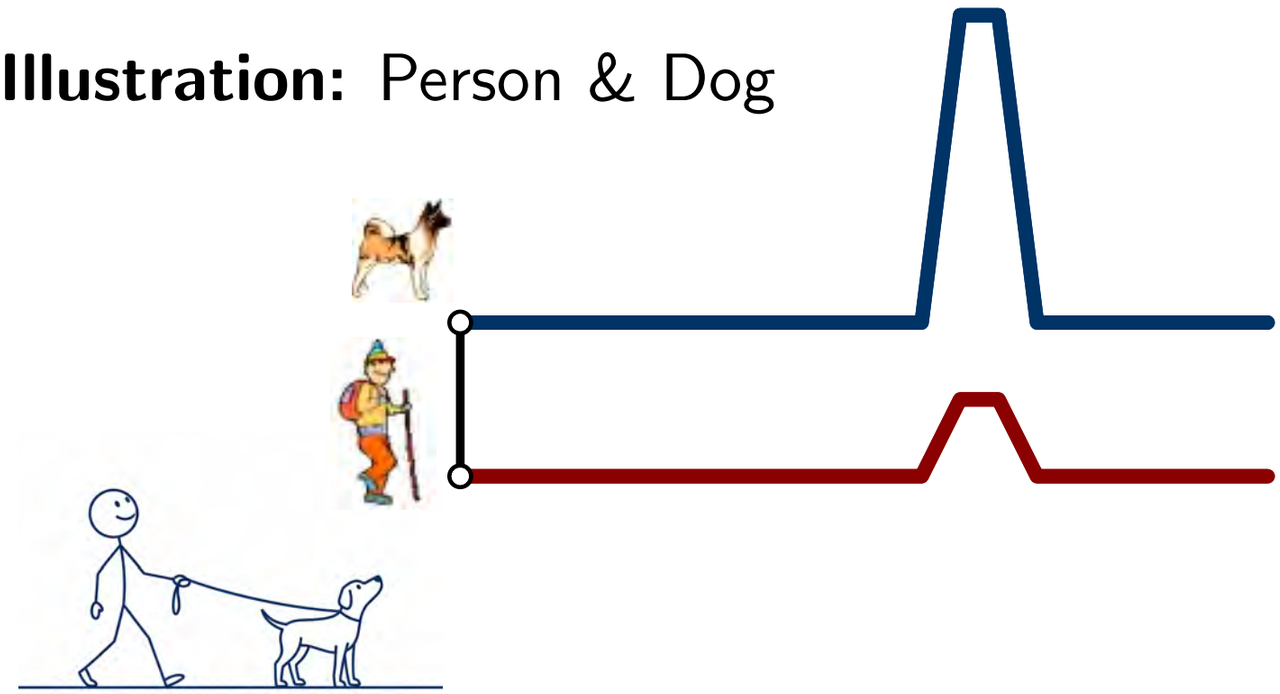

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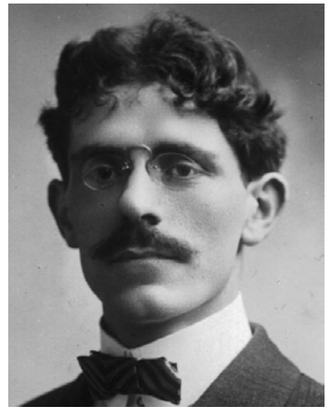

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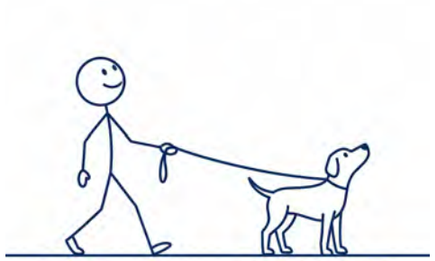

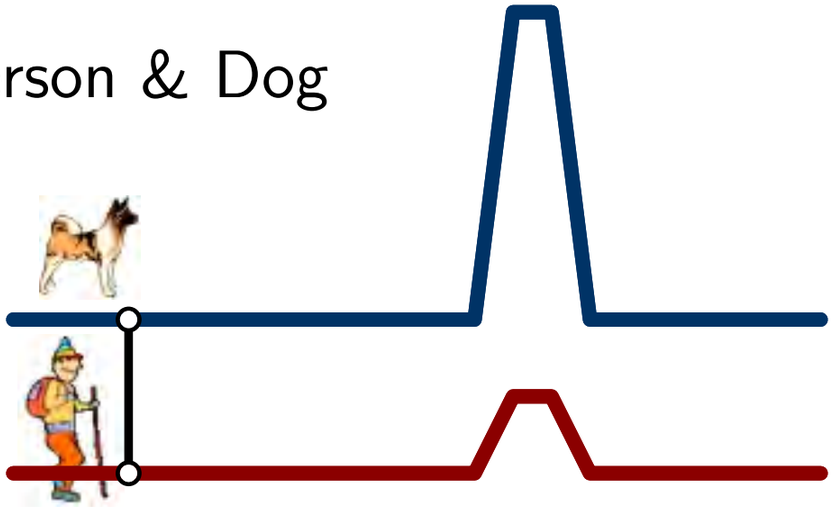

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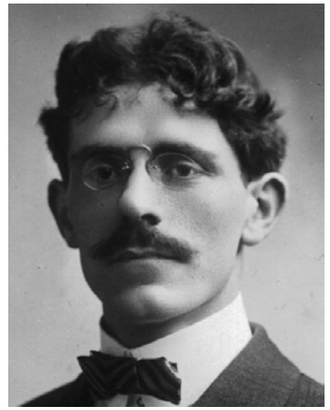

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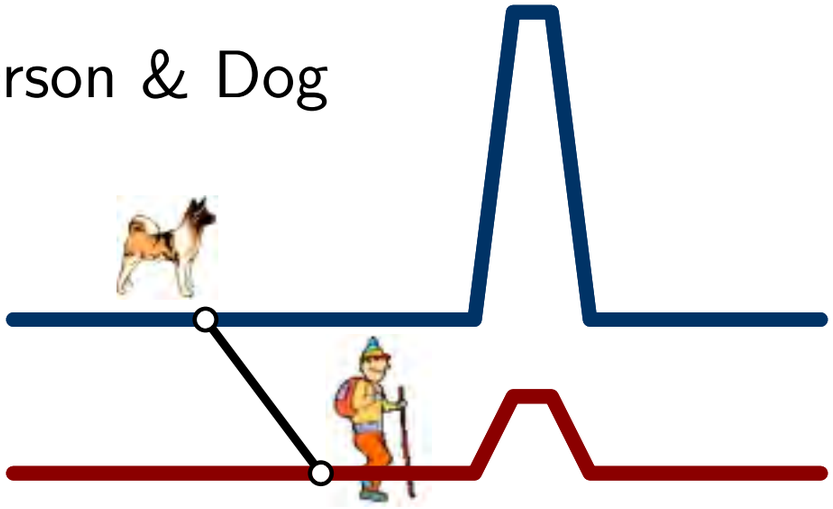

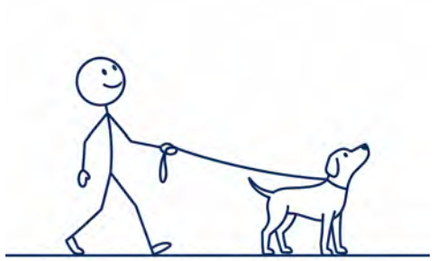

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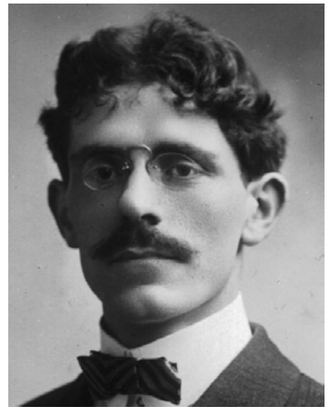

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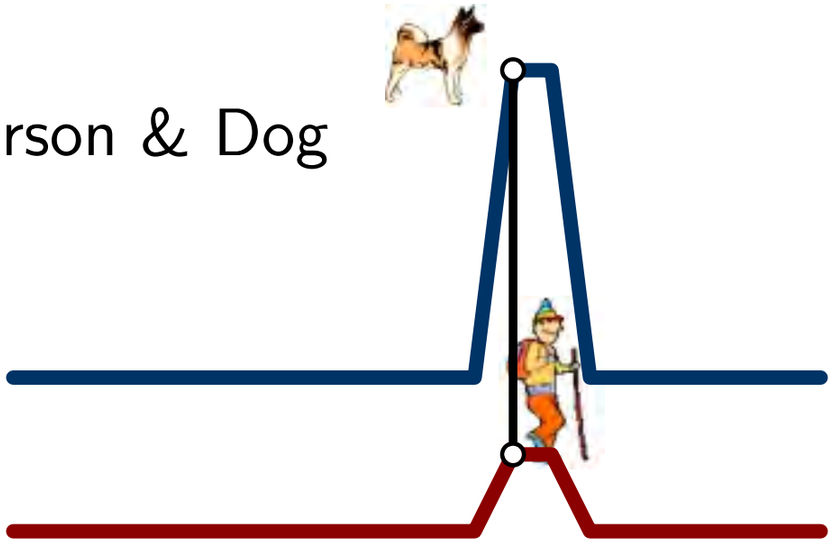

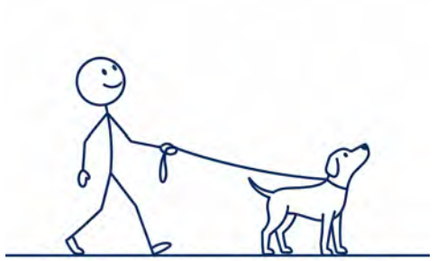

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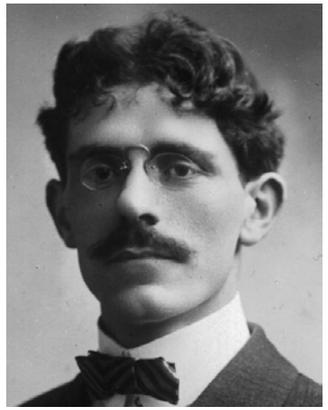

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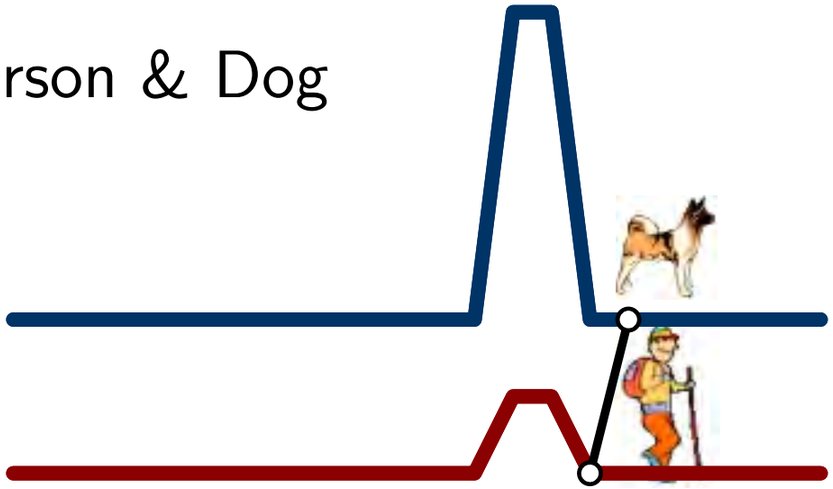

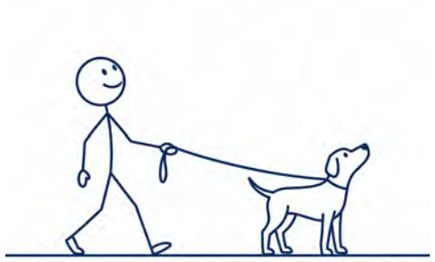

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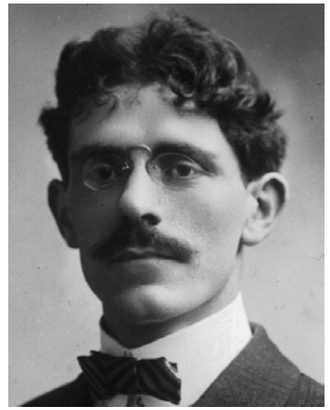

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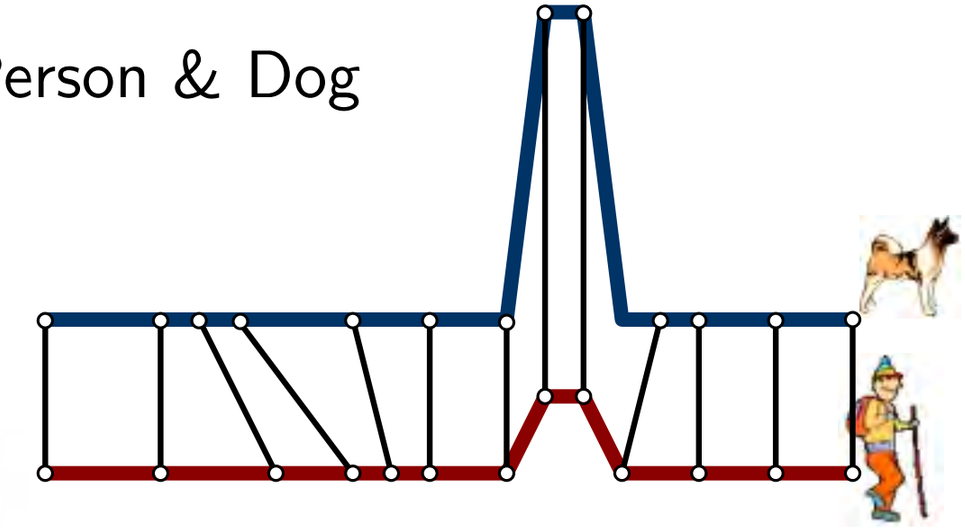

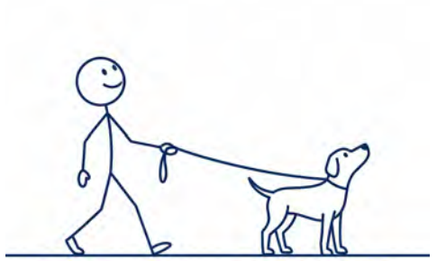

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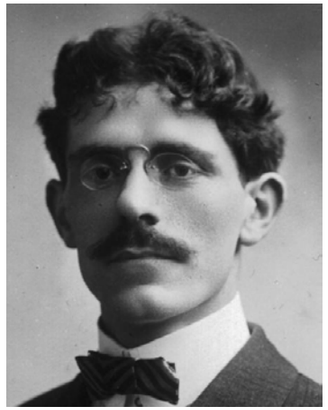

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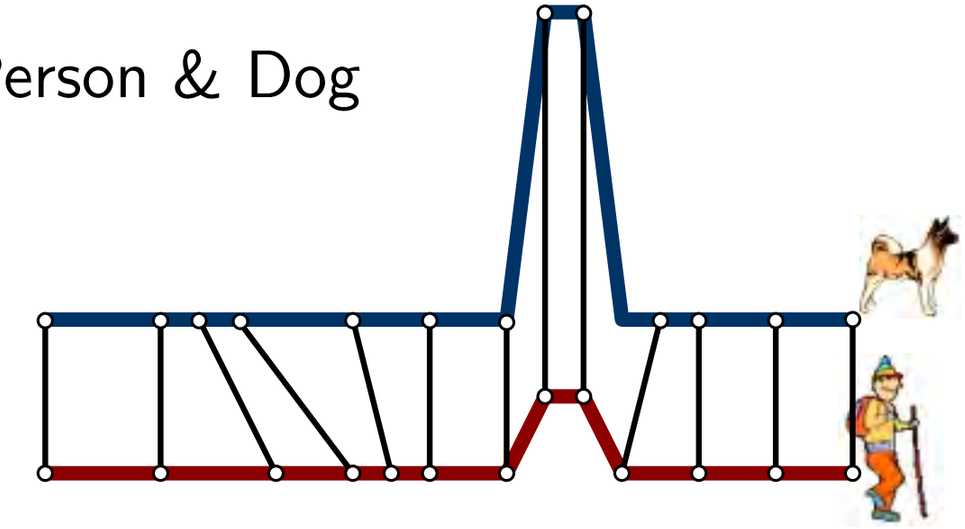

Fréchet distance equals shortest leash length

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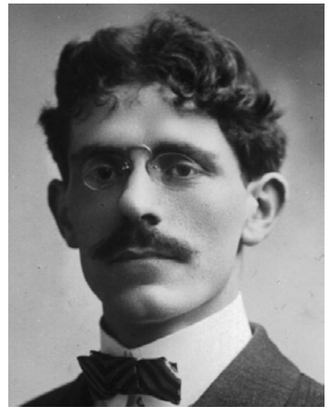

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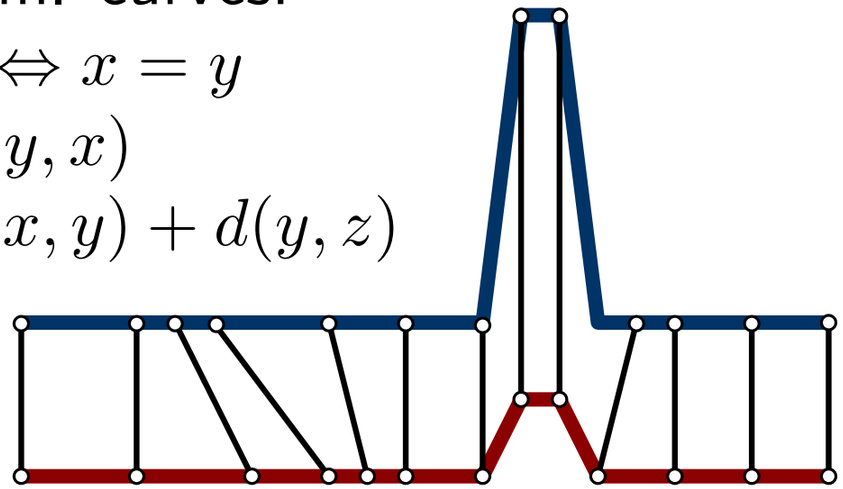
metric for param. curves:

- $d(x,y) = 0 \Leftrightarrow x = y$
- $d(x,y) = d(y,x)$
- $d(x,z) \leq d(x,y) + d(y,z)$



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Research on computing the Fréchet distance

RUB

in 1992 Helmut Alt and Michael Godau presented the first algorithm

A natural metric for curves — Computing the distance for polygonal chains and approximation algorithms

Michael Godau*
godau@leibniz.math.fu-berlin.de
Freie Universität Berlin, FB Mathematik, WE 03
Arnimallee 2–6, D-1000 Berlin 33

Abstract

The often explored problem to approximate a given po
considered from a computational geometric point of view or
model it reasonably we give a natural definition of the di
Furthermore we give algorithms to calculate this distance
chains in the d-dimensional space for arbitrary d. With kno
polynomial time algorithms to approximate polygonal chains
an optimal solution under some constraints. We will show t
by a constant factor worse than the global optimum.

COMPUTING THE FRÉCHET DISTANCE BETWEEN TWO POLYGONAL CURVES *

HELMUT ALT and MICHAEL GODAU
Freie Universität Berlin, Fachbereich Mathematik und Informatik, Takustraße 9
14195 Berlin, Germany
e-mail: alt@inf.fu-berlin.de

Measuring the Resemblance of Polygonal Curves *

(Extended Abstract)

Helmut Alt Michael Godau †

trary dimensions we consider
arametrizations of the curves.
lges an algorithm of runtime
' and Q is developed. Then
net-distance for closed curves,
on derived from the Fréchet-
curve Q.

1 Introduction, Definitions

In many applications, two-dimensional *shapes* are given by the planar curves forming their boundaries. Consequently, a natural problem in shape comparison and recognition is to measure, how much two given curves "resemble each other". Naturally, the first question to be answered, is what metric between curves should be used to reflect the intuitive notion of "resemblance".

Figure 1: Two curves with HAUSDORFF-distance δ.

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and many more papers followed..

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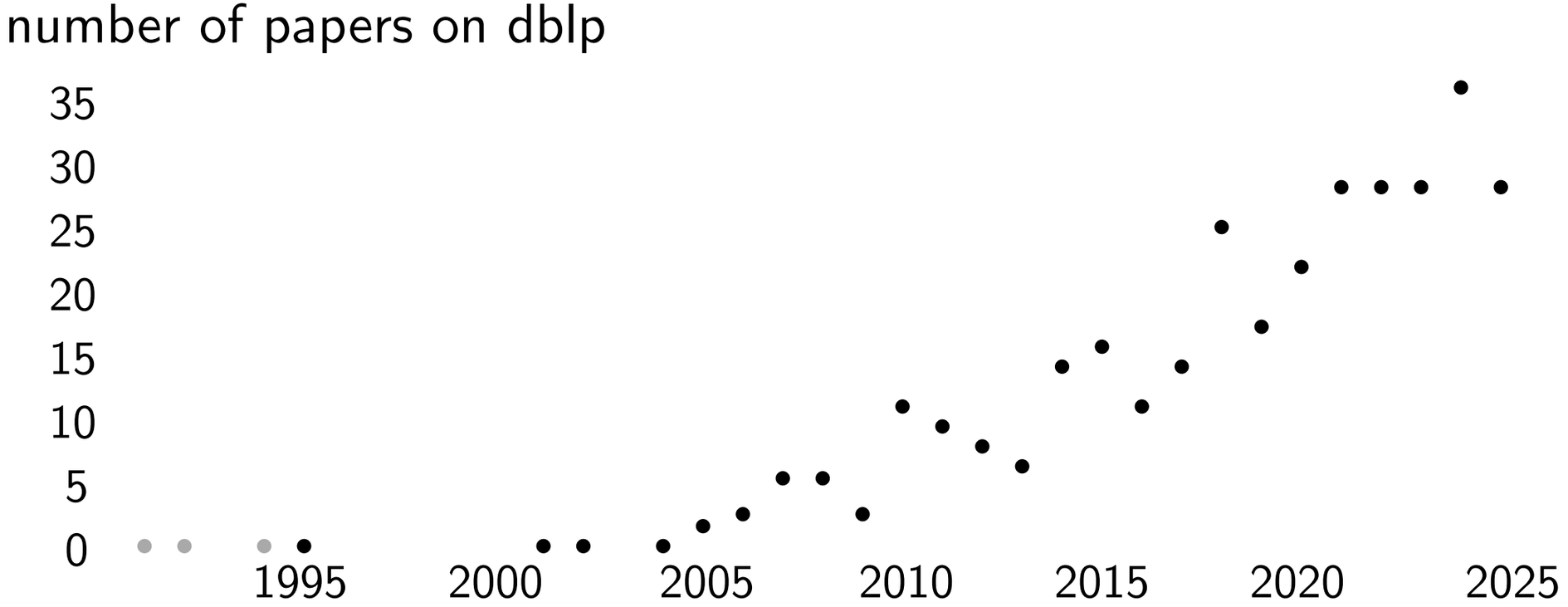
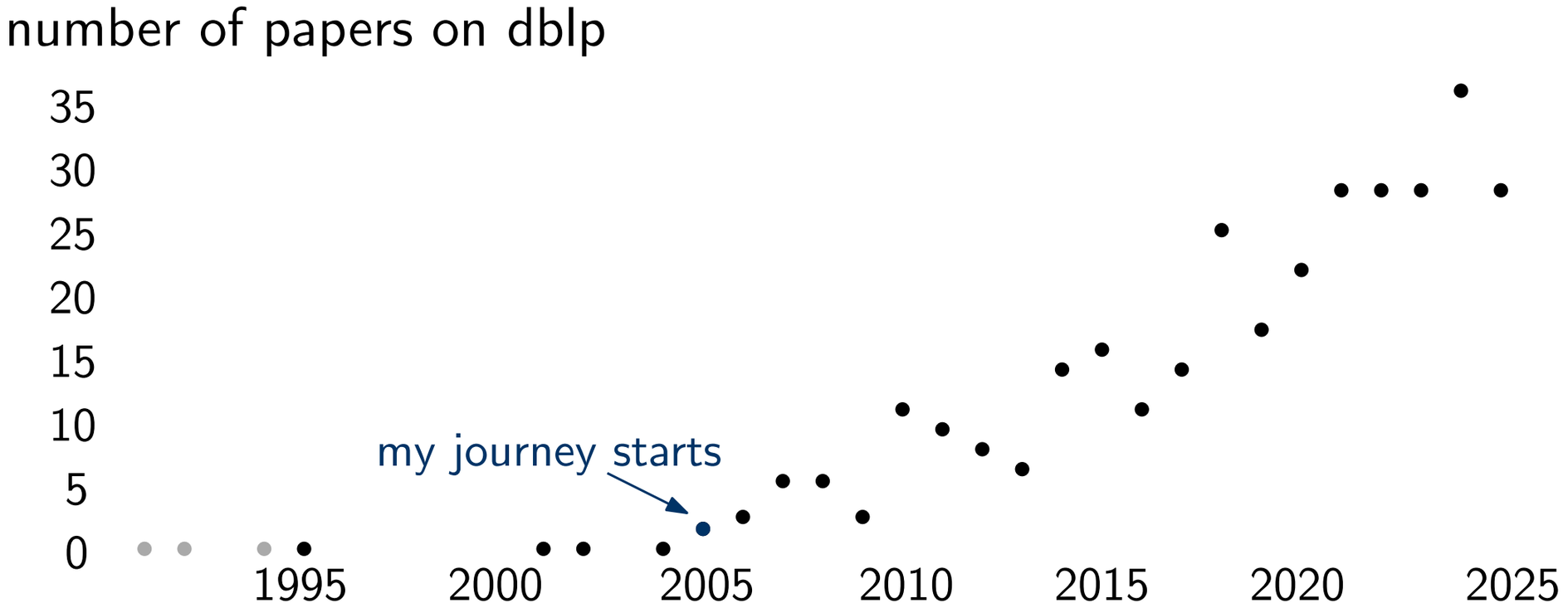
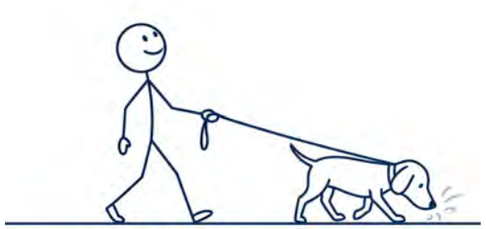


Research on computing the Fréchet distance

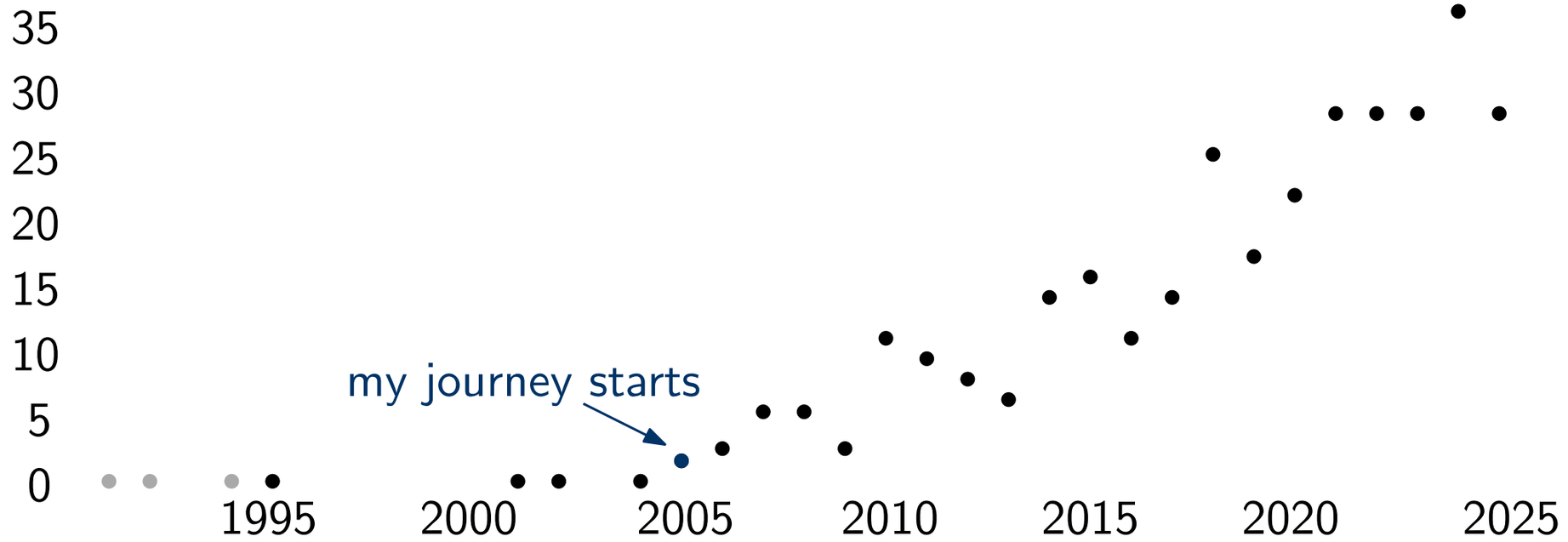




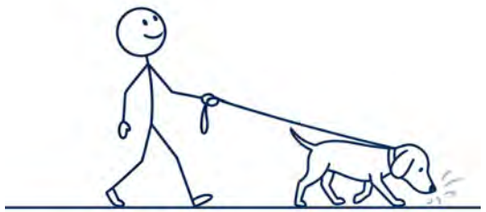
Research on computing the Fréchet distance

RUB

number of papers on dblp



why so popular?



Research on computing the Fréchet distance

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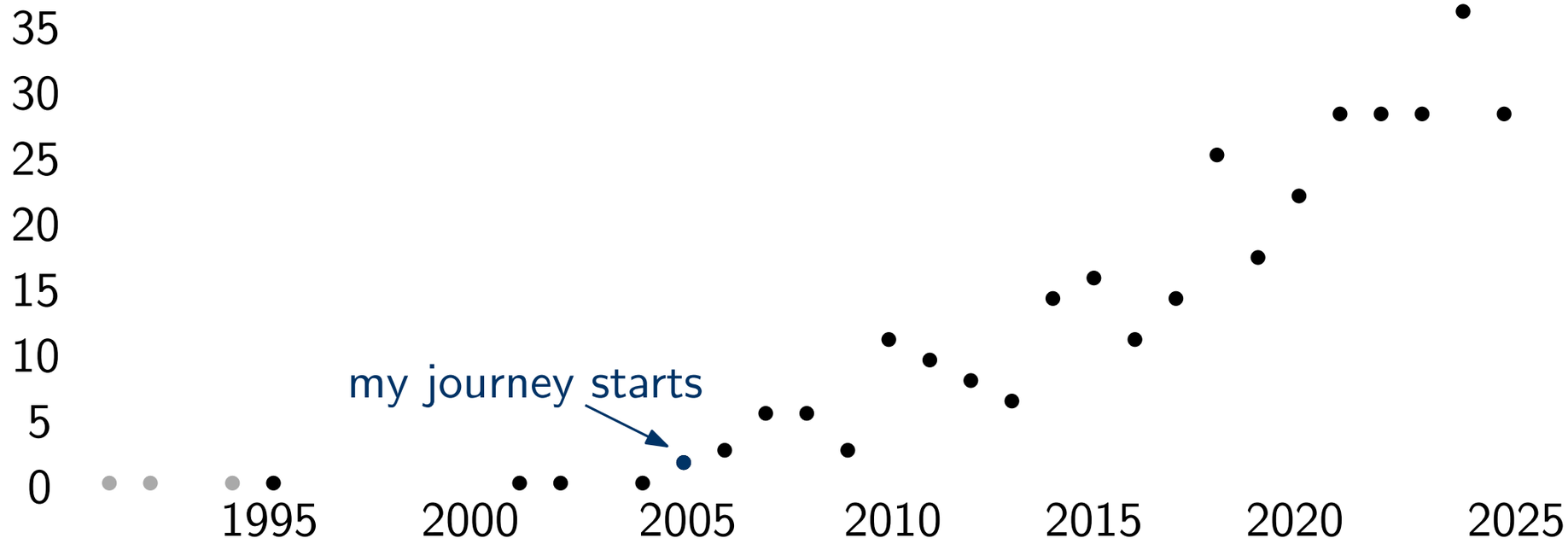

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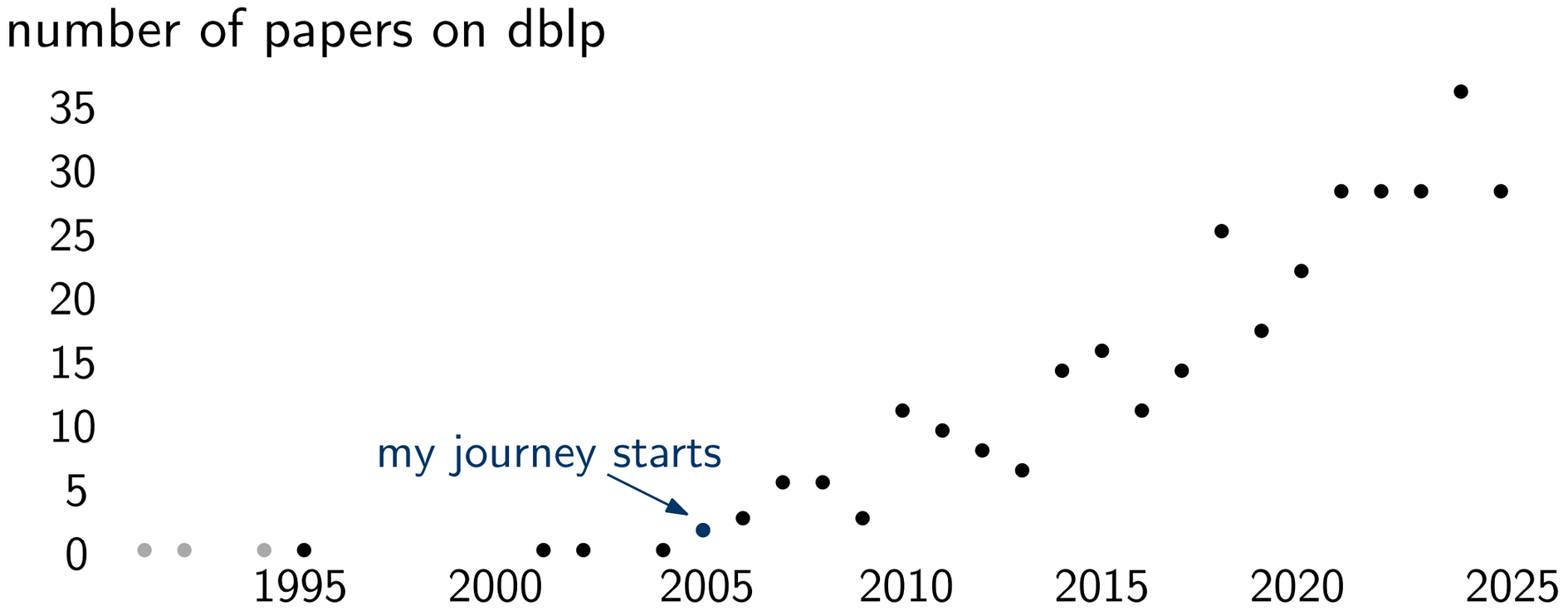
Research on computing the Fréchet distance

RUB



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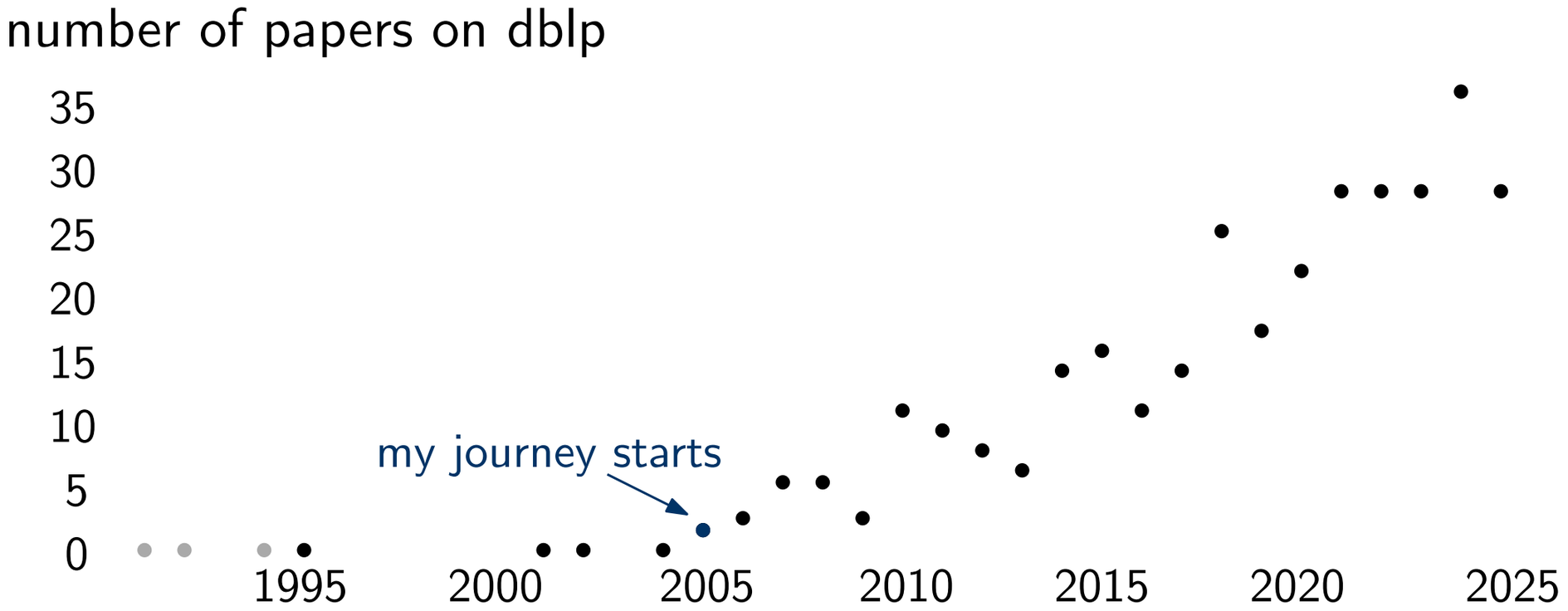
Plan for this talk:

- some personal highlights
- open problems

Research on computing the Fréchet distance

RUB



why so popular? because it's *natural*, *versatile* and *challenging* to compute!

Plan for this talk:

- some personal highlights
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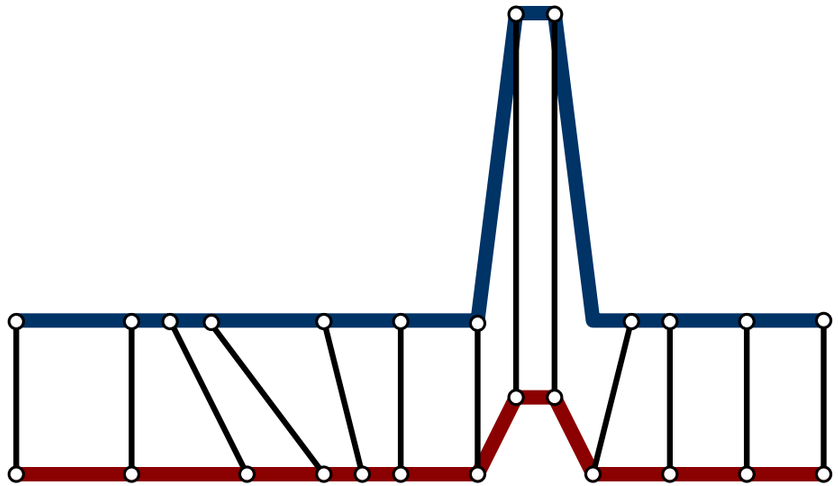
Disclaimer:
only continuous (not discrete) Fréchet distance

Algorithm of Alt & Godau

RUB

Let P, Q be two polygonal curves of size n.

- **decision problem:** Is $d_F(P, Q) \leq \varepsilon$?
- **computation problem:** What is $d_F(P, Q)$?



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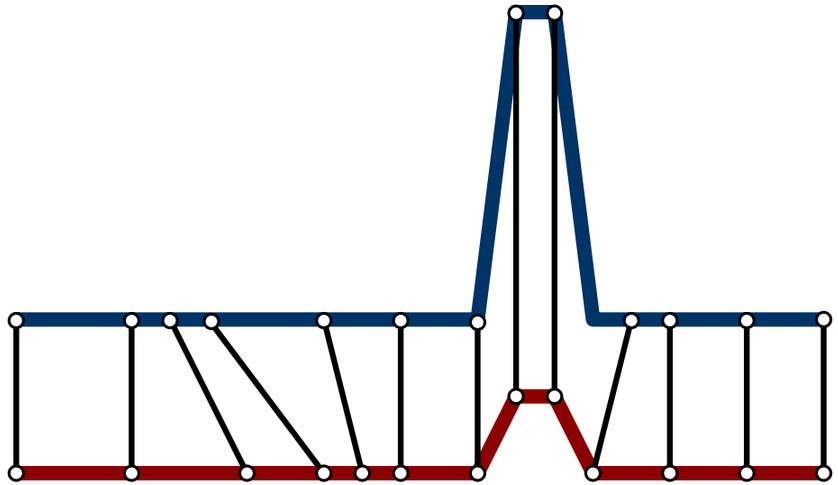
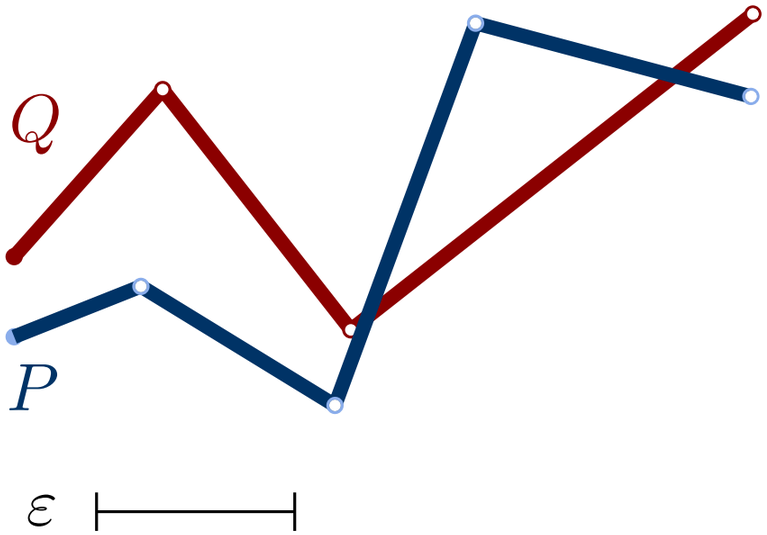
use the *Free Space Diagram* for the decision problem

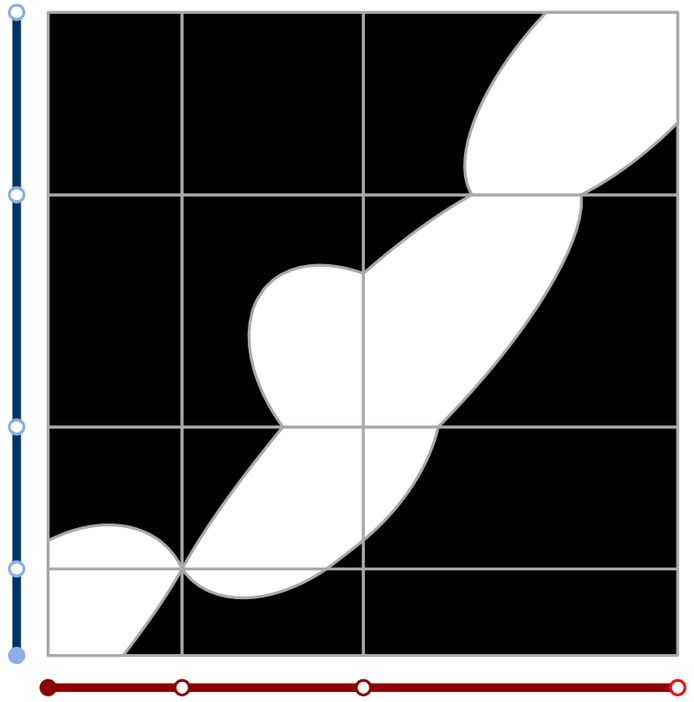


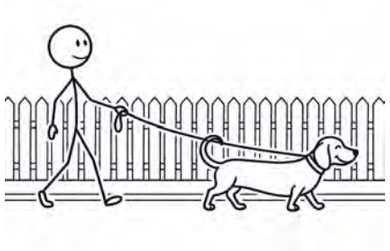
Free Space Diagram

Let $P, Q\colon [0, n] \to \mathbb{R}^d$ polygonal curves, and let $\varepsilon > 0$.

Define $F_\varepsilon(P, Q) := \{(s, t) \mid d(P(s), Q(t)) \leq \varepsilon\}$







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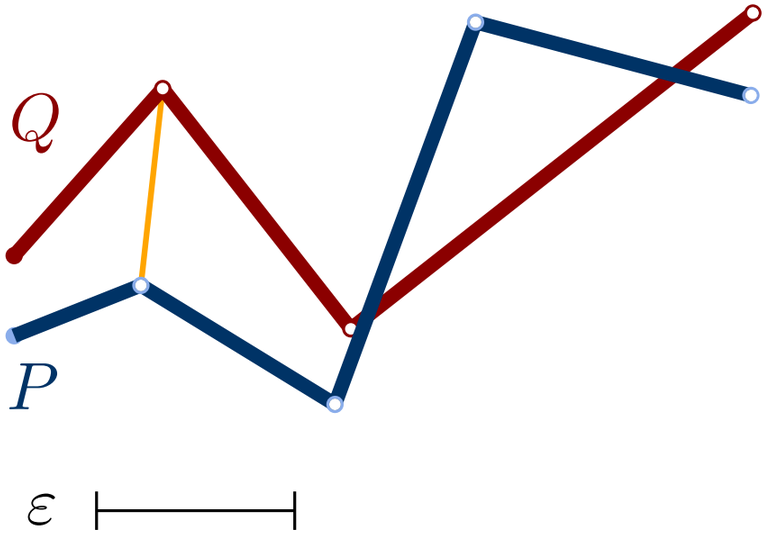

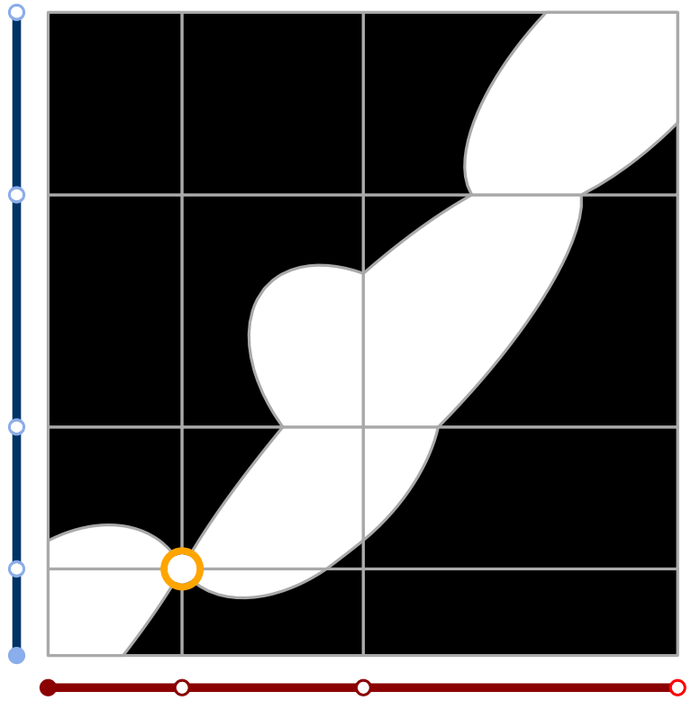

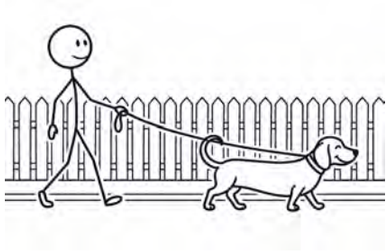

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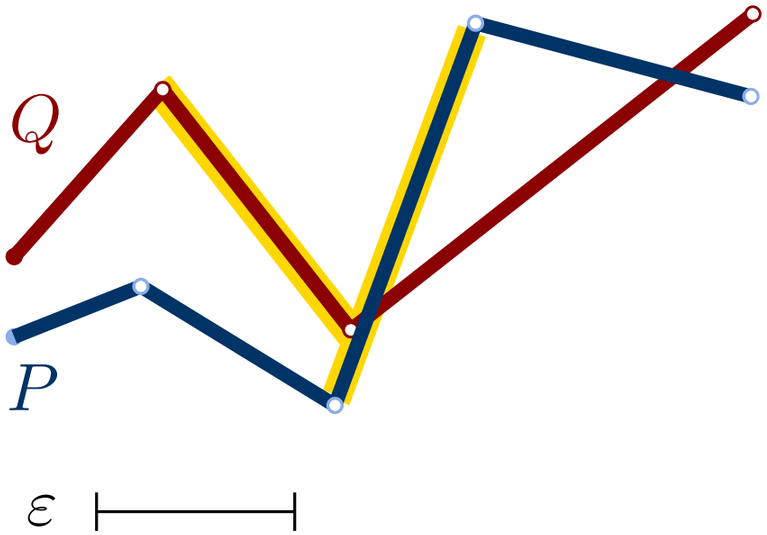
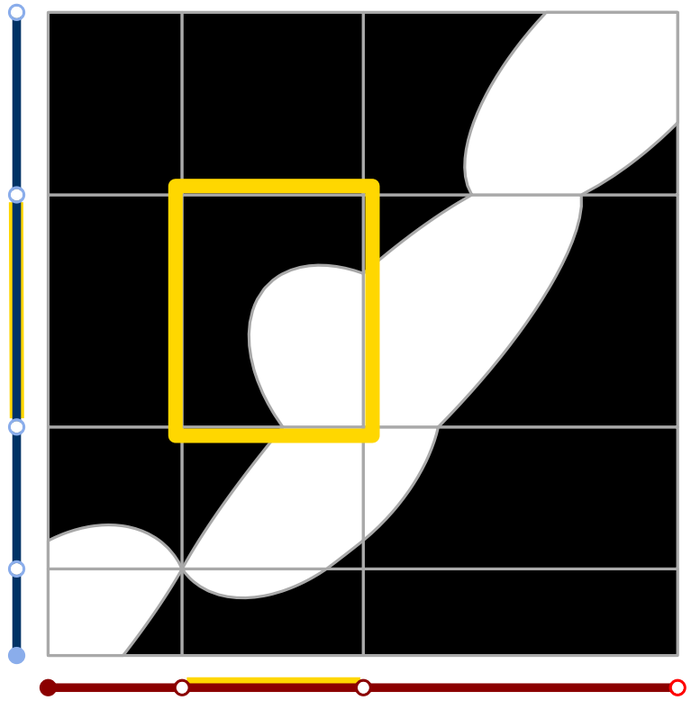
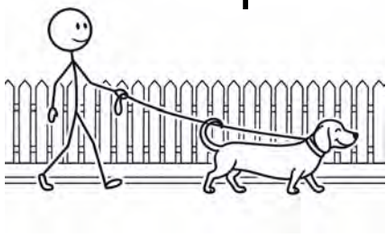




Cell: free space of two segments of the curves



Free Space Diagram

Let $P, Q\colon [0, n] \to \mathbb{R}^d$ polygonal curves, and let $\varepsilon > 0$.

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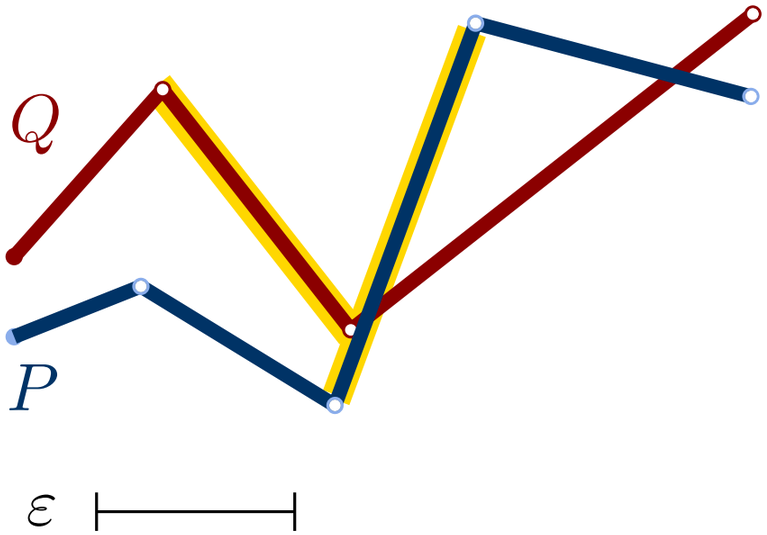
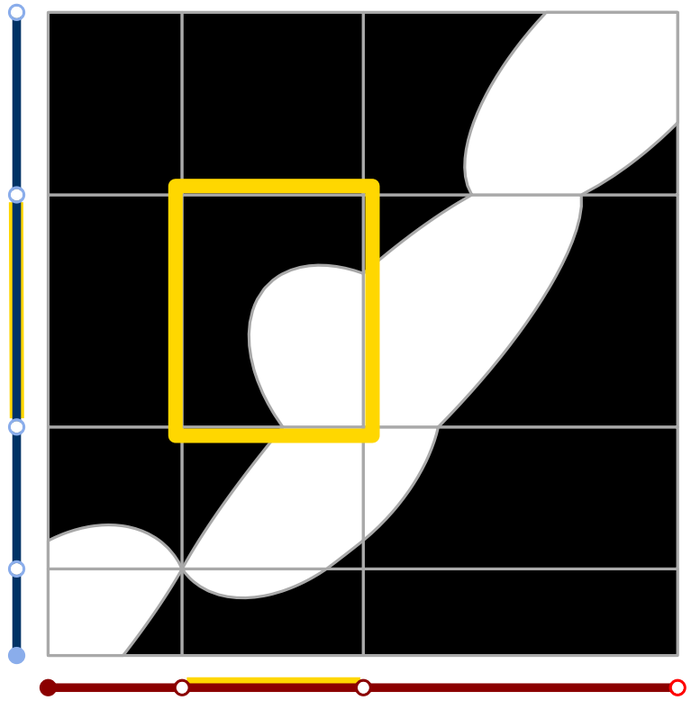
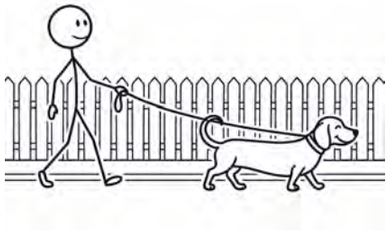




Lemma 1: Cells of the free space are convex.



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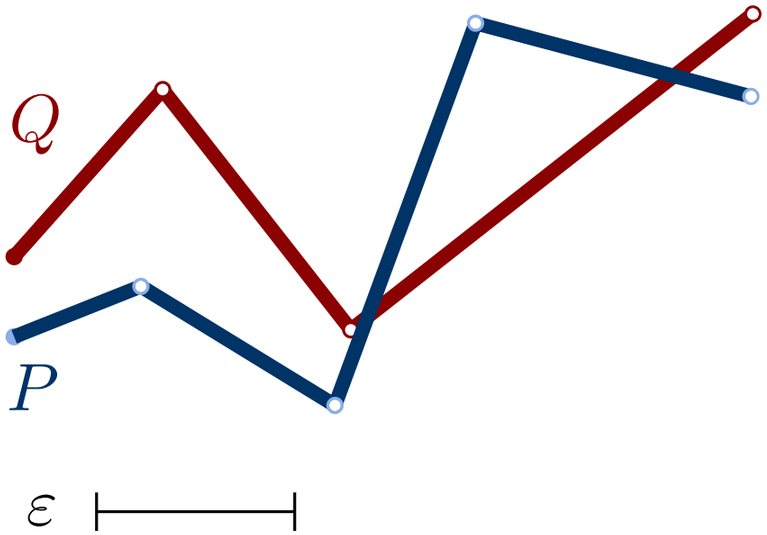
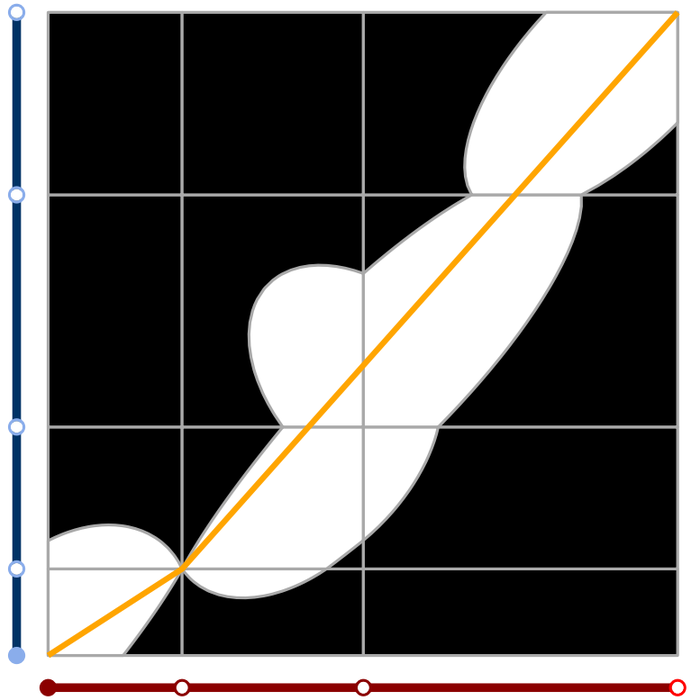




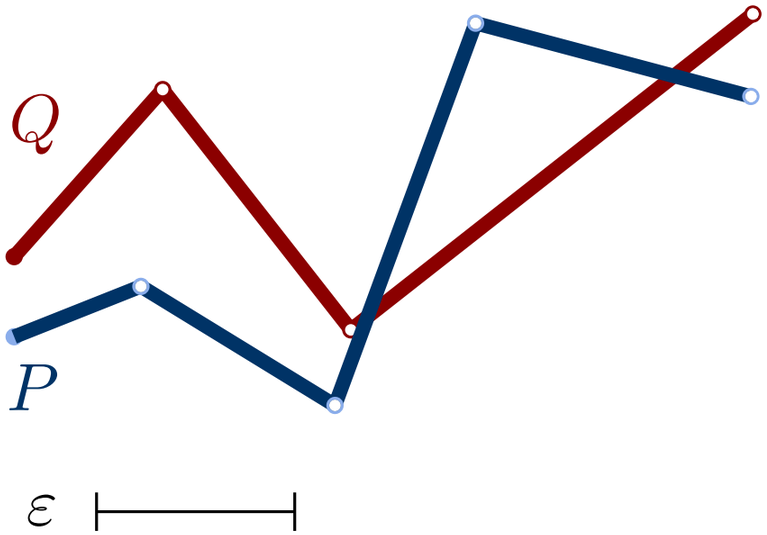
Lemma 1: Cells of the free space are convex.

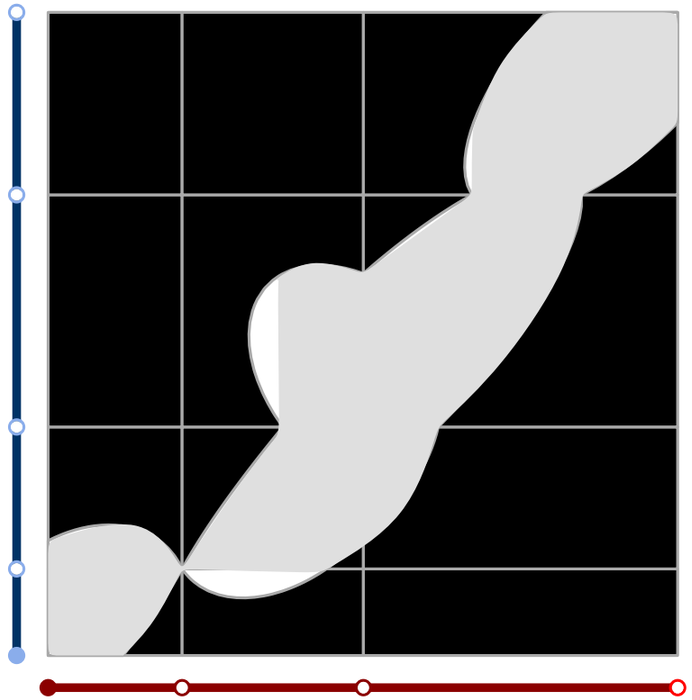
Lemma 2: $d_F(P, Q) \leq \varepsilon \Leftrightarrow$ there exists a monotone path from $(0, 0)$ to (n, n) in the free space $F_\varepsilon(P, Q)$.

Free Space Diagram

Let $P, Q\colon [0, n] \to \mathbb{R}^d$ polygonal curves, and let $\varepsilon > 0$.

Define $F_\varepsilon(P, Q) := \{(s, t) \mid d(P(s), Q(t)) \leq \varepsilon\}$



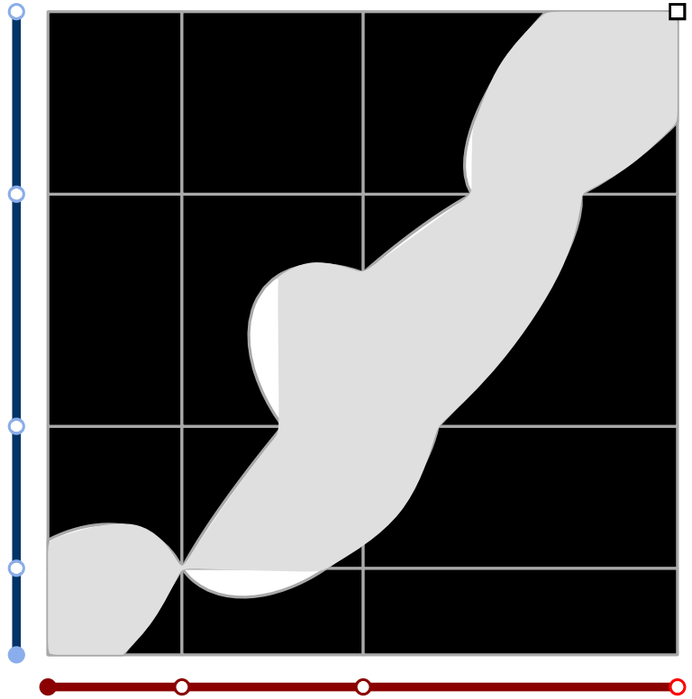


reachable free space: subset of $F_\epsilon(P, Q)$ that is reachable by a monotone path from $(0, 0)$

Decision Algorithm

Input: Polygonal curves P, Q, and $\varepsilon > 0$
Output: Is $d_F(P, Q) \leq \varepsilon$

compute lower and left reachable boundaries of the diagram
for $i \leftarrow 1$ **to** n **do**
  for $j \leftarrow 1$ **to** n **do**
    compute upper and right reachable boundaries of cell (i, j)
output **true** if (n, n) reachable, else **false**



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RUB

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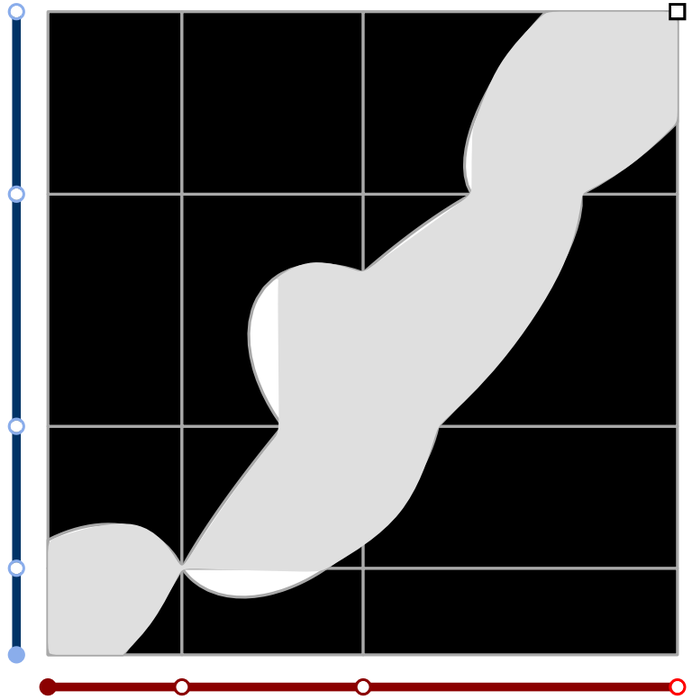
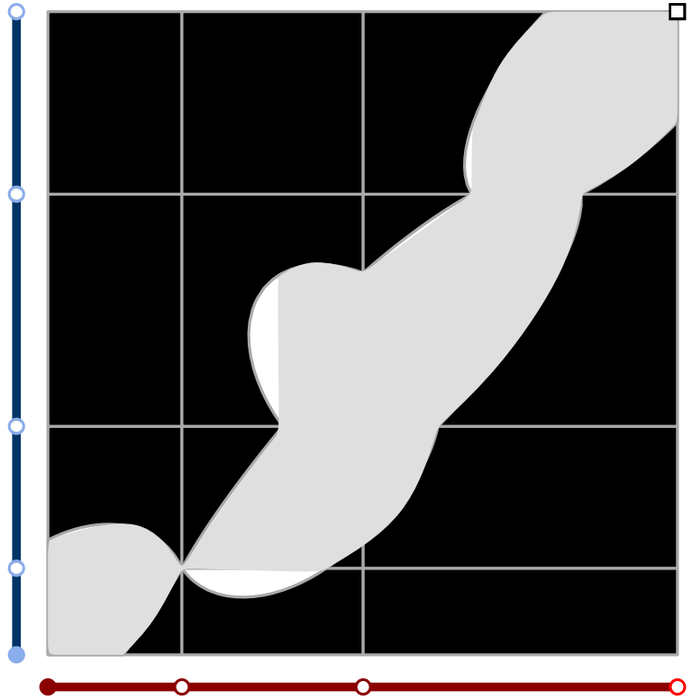


Theorem (Alt, Godau)
For two polygonal curves P, Q of size n and $\varepsilon > 0$, $d_F(P, Q) \leq \varepsilon$ can be decided in $O(n^2)$ time and $O(n)$ space.

Computation Algorithm

RUB

Input: Polygonal curves P, Q
Output: $d_F(P, Q)$
determine all critical values
sort the critical values
do a **binary search** over the values,
solving the decision problem in each step



critical values:
values of ε where reachability in the free space changes

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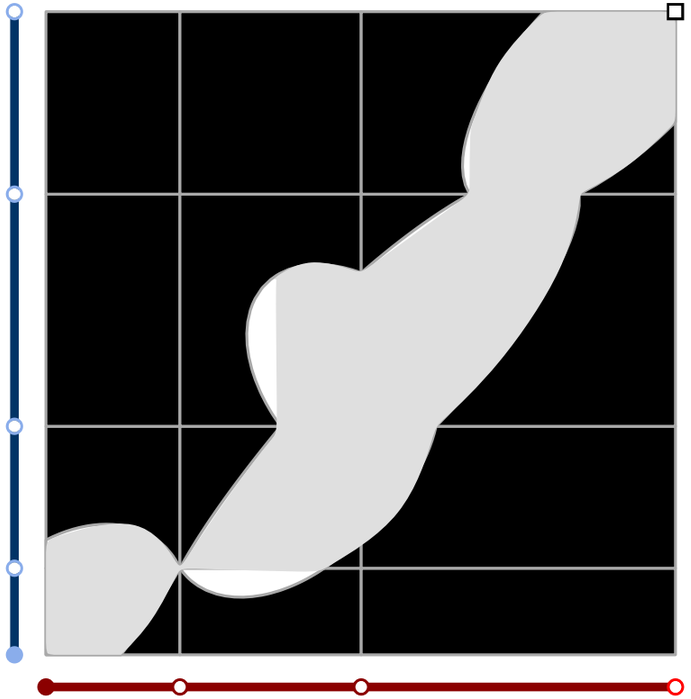
Runtime: $O(n^3 \log n)$



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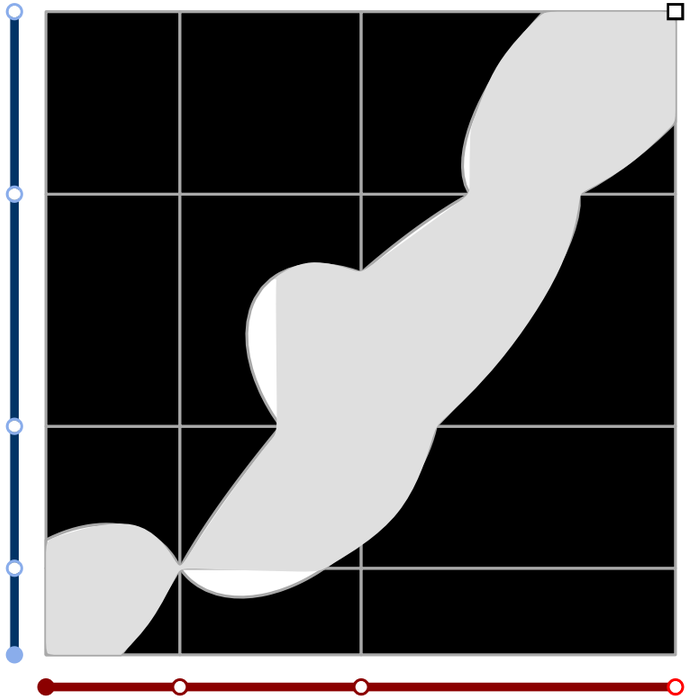

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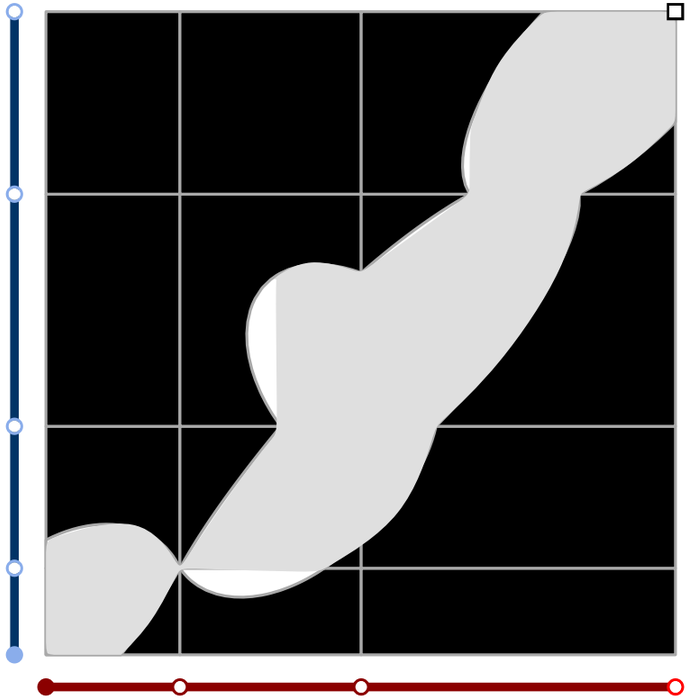

critical values:
values of ε where reachability in the free space changes

Theorem (Alt, Godau '95)
For two polygonal curves P, Q of size n, one can

- decide $d_F(P, Q) \leq \varepsilon$ in $O(n^2)$ time
- compute $d_F(P, Q)$ in $O(n^2 \log n)$ time

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Questions:

- faster?
- implementation?
- generalizations?
- applications?

Computation Algorithm

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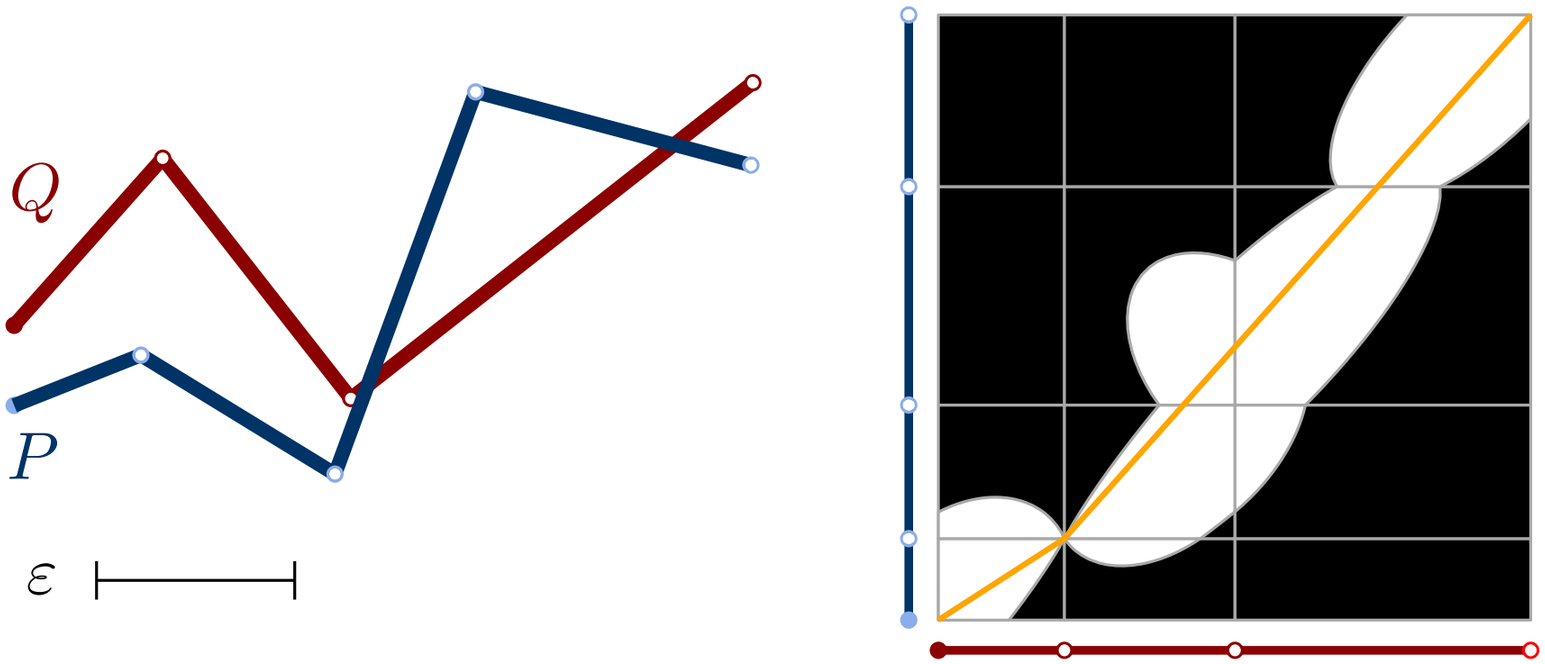
Breaking the quadratic runtime

RUB

Can the Fréchet distance be computed in subquadratic time?



Bottleneck: size of the free space diagram

Breaking the quadratic runtime

RUB

Can the Fréchet distance be computed in subquadratic time?

- conditional lower bound by Bringmann (2014)
- slightly faster algorithm by Buchin et al. (2014)
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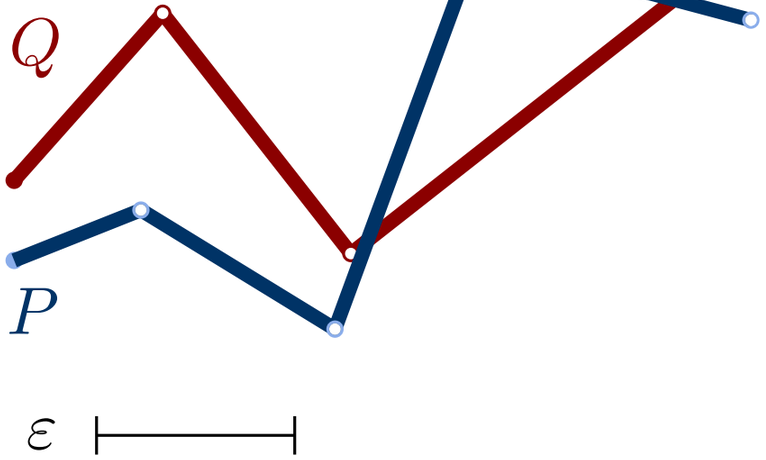

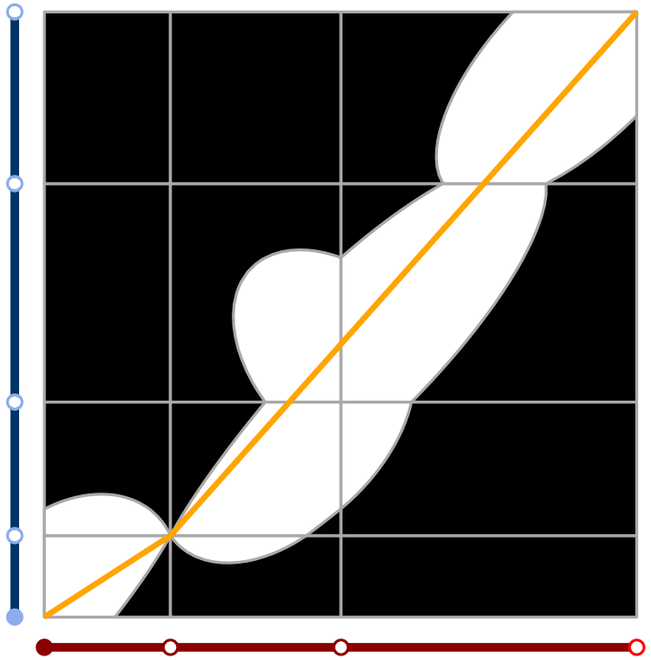

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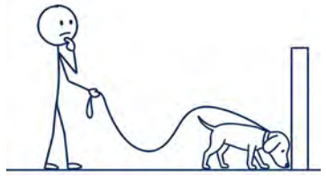
Reduction from k-CNF SAT

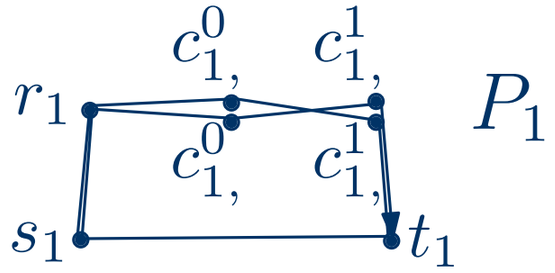
Given CNF-SAT formula F with N variables and M clauses.

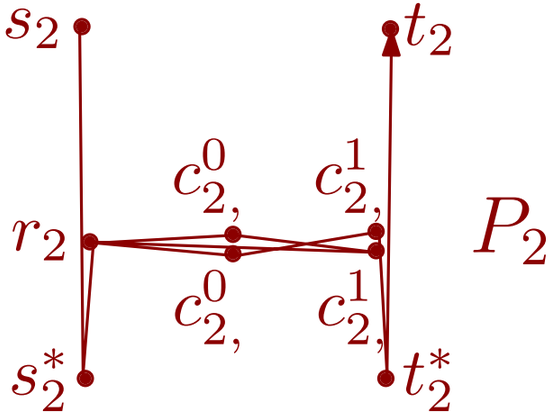
Construct two curves P_1, P_2 such that

$$d_F(P_1, P_2) \leq 1 \quad \Leftrightarrow \quad F \text{ is satisfiable.}$$

Hence an $O(n^{2-\delta})$ algorithm for d_F would imply an $O(M^2 2^{(1-\delta/2)N})$ SAT algorithm.







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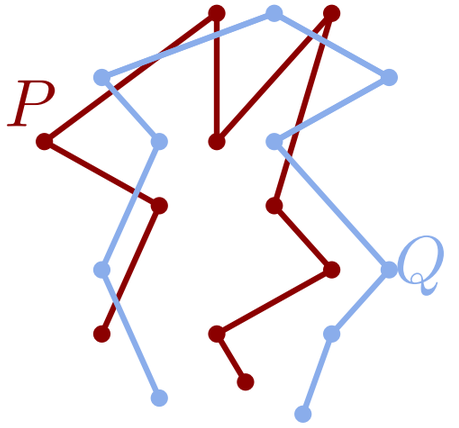
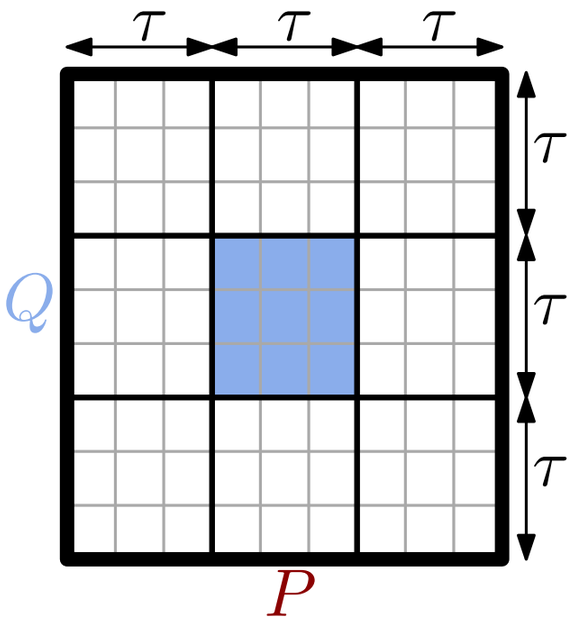
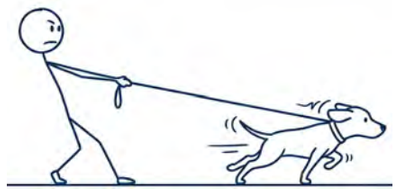
Pre-processing to speed up the computation (4 Russians trick)



elementary boxes





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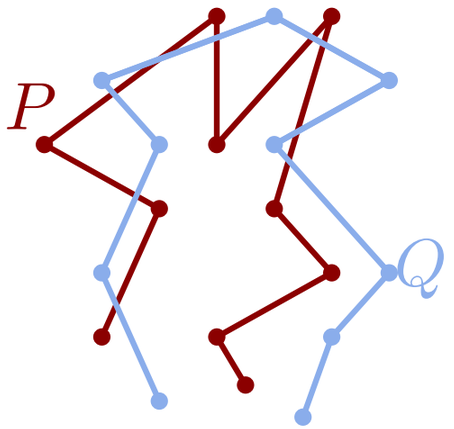

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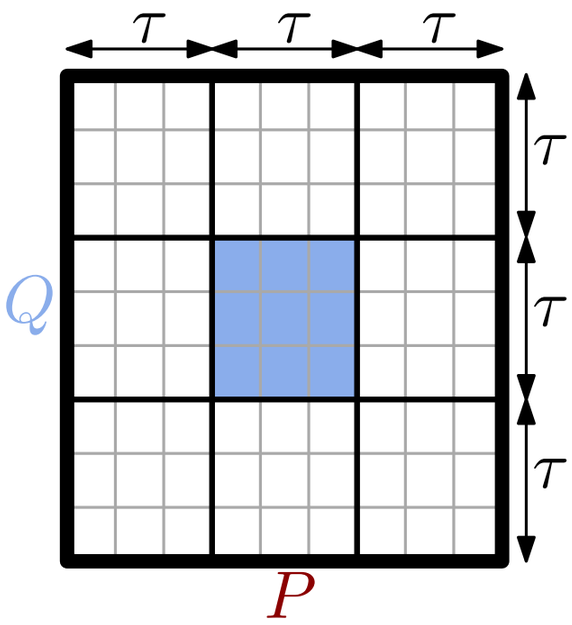
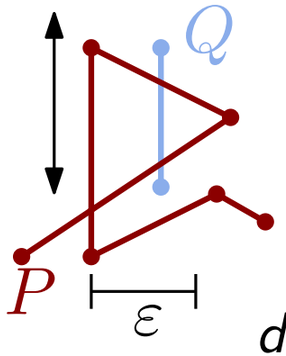
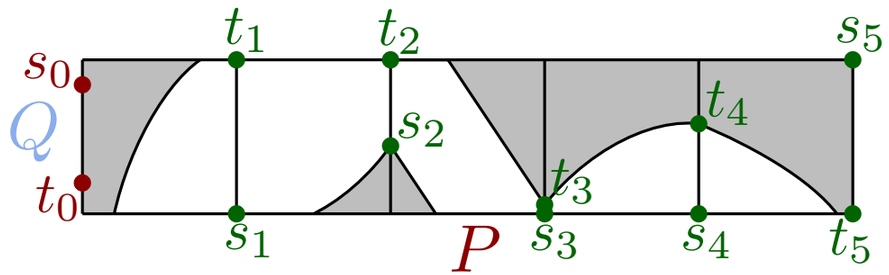
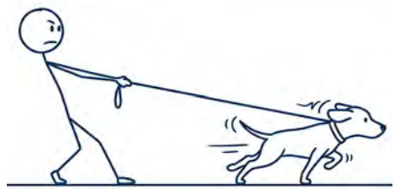






door orders

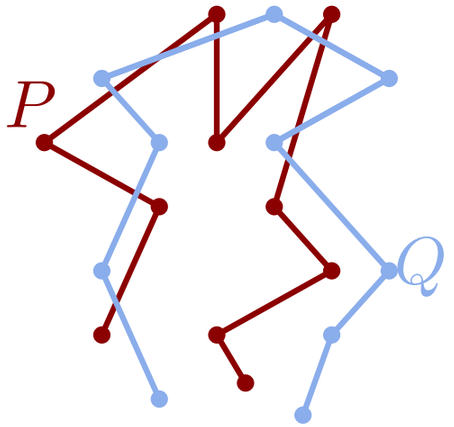


Breaking the quadratic runtime

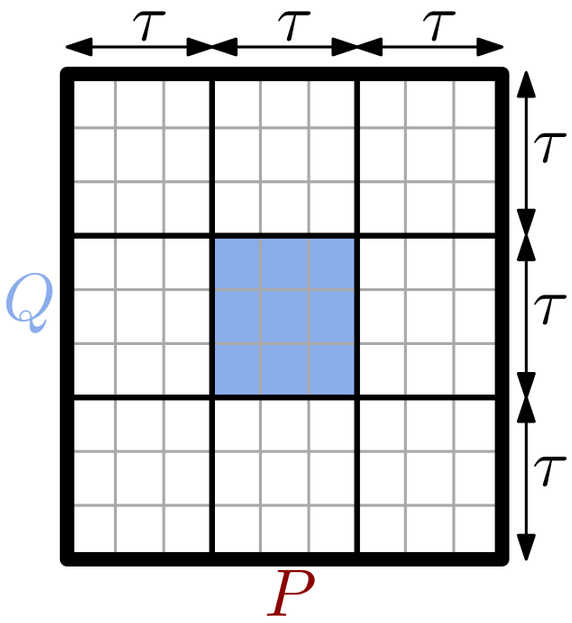
Can the Fréchet distance be computed in subquadratic time?

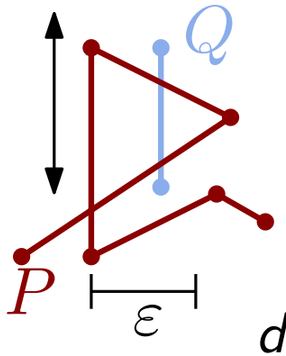
- conditional lower bound by Bringmann (2014)
- **slightly faster algorithm by Buchin et al. (2014)**
- weakly subquadratic time by Cheng and Huang (2025)

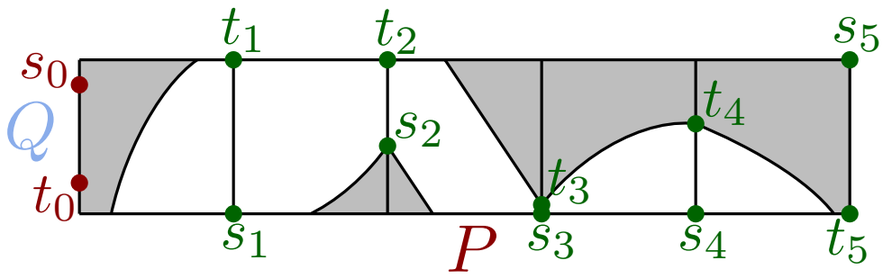
Pre-processing to speed up the computation (4 Russians trick)



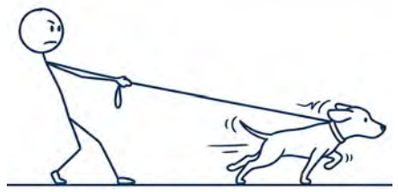
elementary boxes

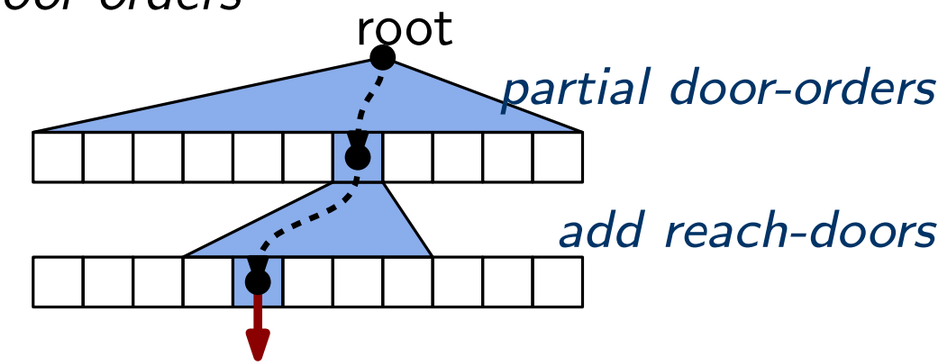






door orders





Breaking the quadratic runtime

RUB

Can the Fréchet distance be computed in subquadratic time?

- conditional lower bound by Bringmann (2014)
- slightly faster algorithm by Buchin et al. (2014)
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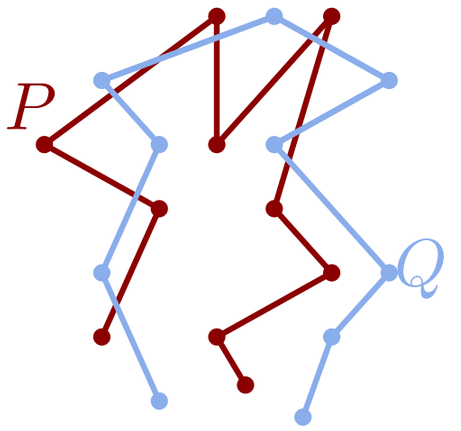
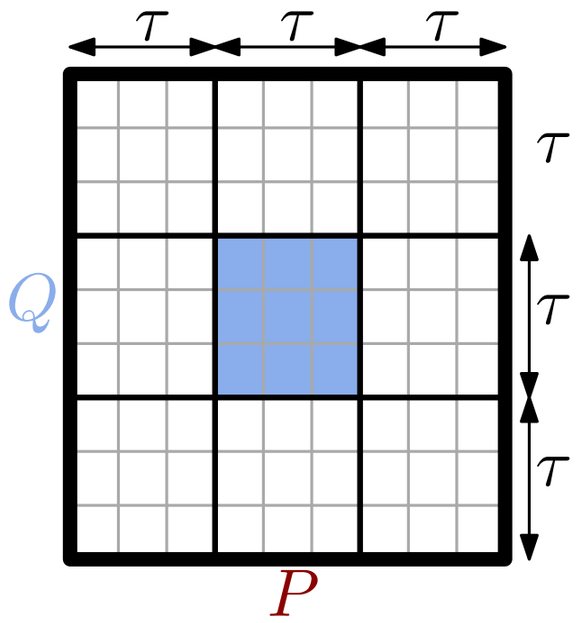
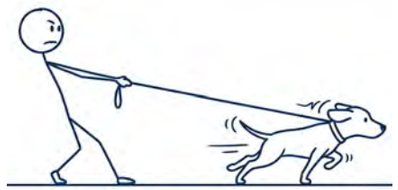
Pre-processing to speed up the computation (4 Russians trick)



elementary boxes



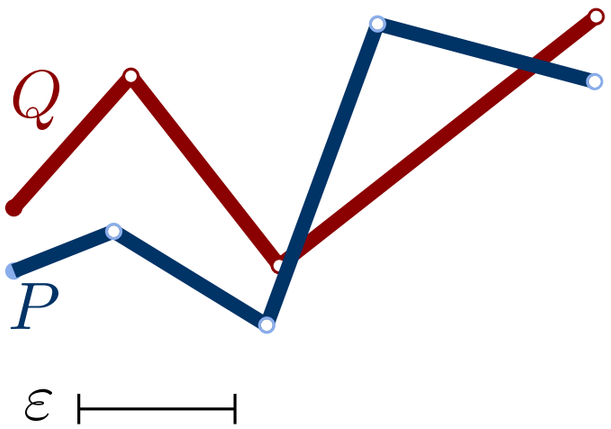
use algebraic geometric tools to show only a subquadratic number of distinct encodings

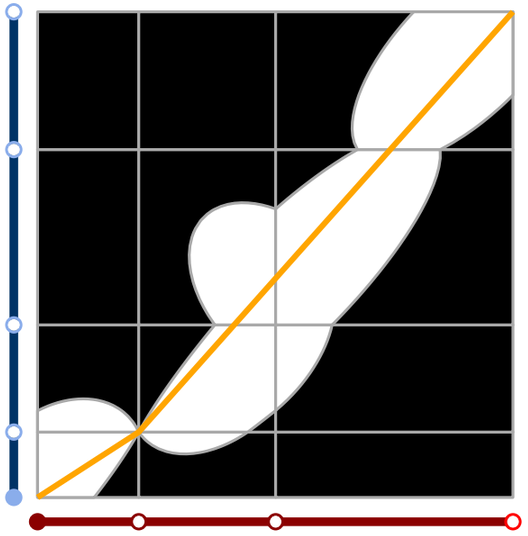


Realistic inputs

RUB

Can the Fréchet distance be computed faster for certain inputs?





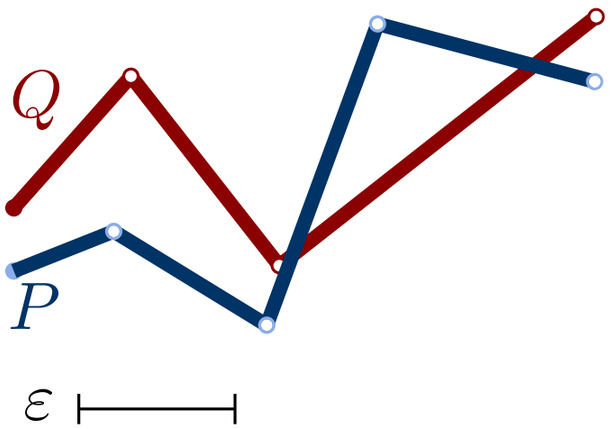
Bottleneck: size of the free space diagram

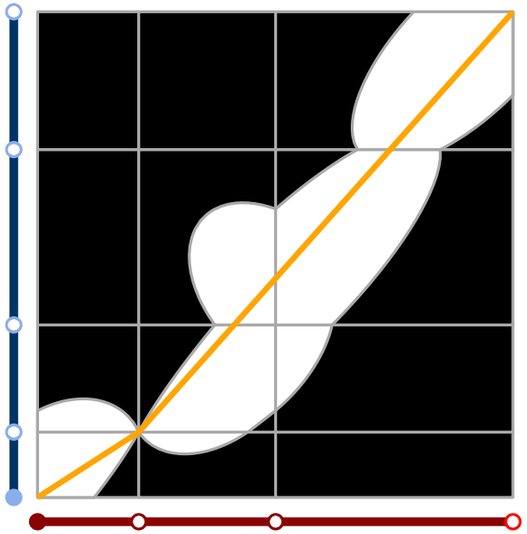
Realistic inputs

RUB

Can the Fréchet distance be computed faster for certain inputs?

- κ-straight, κ-bounded (Alt, Knauer, Wenk, 2004)
- backbone curves (Aronov et al. 2006)
- c-packed curves (Driemel, Har-Peled, Wenk, 2012),





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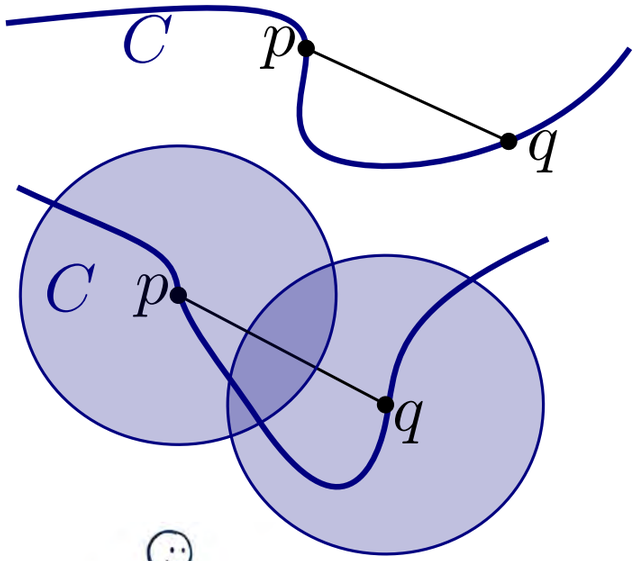
κ-straight

for p, q on curve C: $|C[p,q]| \leq \kappa||p-q||$

κ-bounded

for p, q on curve C: $C[p,q] \subseteq B(p,r) \cup B(q,r)$,
where $r = \kappa||p-q||/2$

for these curves Fréchet distance can be
bounded by $\kappa + 1$ times Hausdorff distance



Realistic inputs

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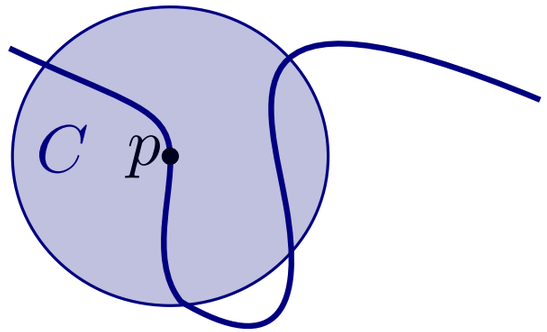
- **c-packed curves (Driemel, Har-Peled, Wenk, 2012)**

c-packed
for any p: $|C \cap B(p,r)| \leq cr$

- simplify curves (c-packed remains $6c$-packed, complexity of fsd reduced)
- approximately decide Fréchet distance between simplifications



Realistic inputs

RUB

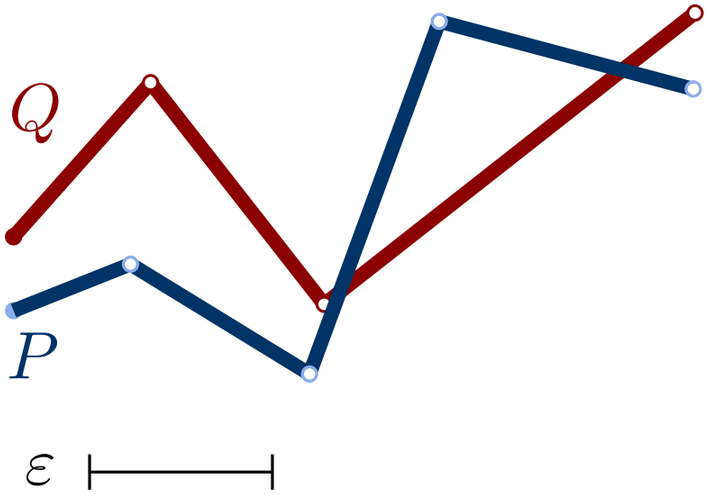
Can the Fréchet distance be computed faster for certain inputs?

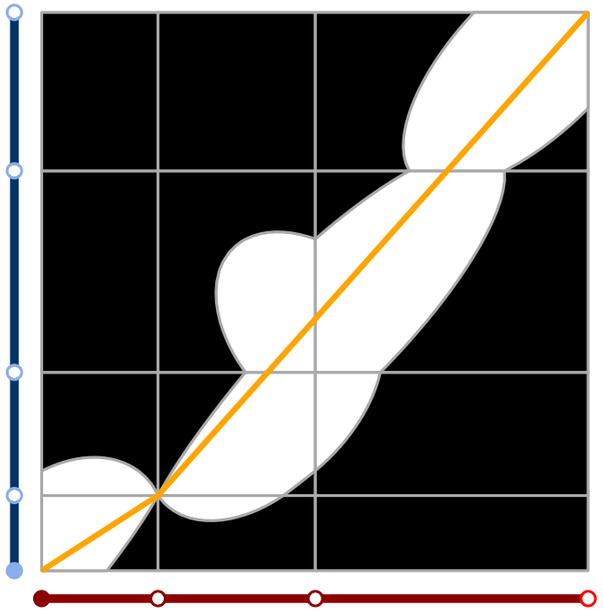
- κ-straight, κ-bounded (Alt, Knauer, Wenk, 2004)
- backbone curves (Aronov et al. 2006)
- c-packed curves (Driemel, Har-Peled, Wenk, 2012),
- long edges (Gudmundsson, Mirzanezhad, Mohades, Wenk, 2019)
- one curve c-packed (Gudmundsson, Mai, Wong, 2024),
  (Conradi, van der Hoog, van der Horst, Ophelders, 2026)
- imbalanced (Blank, 2026)

Approximation

RUB

Can the Fréchet distance be approximated faster?



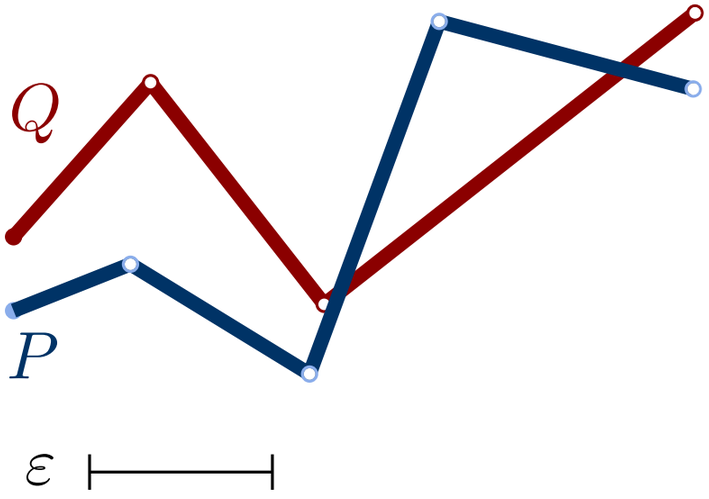


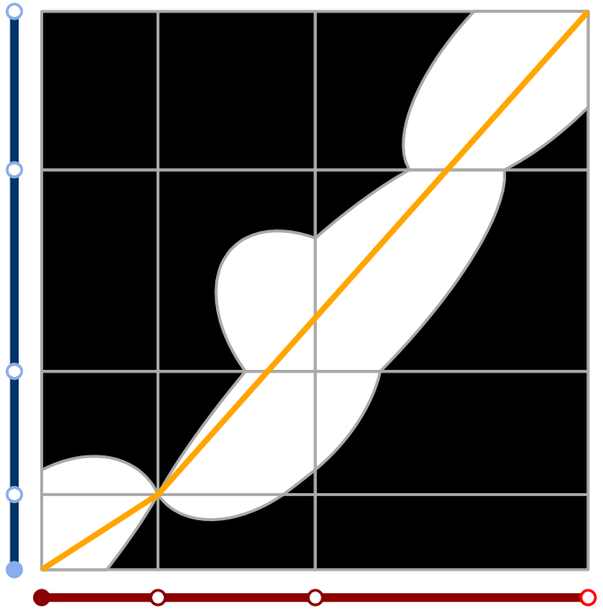
Approximation

RUB

Can the Fréchet distance be approximated faster?

- no 3-approx in subquadratic time (Buchin, Ophelders, Speckmann, 2019)
- $O(\alpha)$-approx for $\alpha \in [\sqrt{n}, n]$ in $O((n^3/\alpha^2)\log^3 n)$ (Colombe & Fox, 2021)
- $O(\alpha)$-approx for $\alpha \in [1, n]$ in $O((n + mn/\alpha)\log^3 n)$ (van der Horst, van Kreveld, Ophelders & Speckmann, 2023)
- $O(\alpha)$-approx for $\alpha \in [1, n]$ in $O((n + mn/\alpha)\log^2 n)$ (van der Horst, Ophelders, '24)
- $(7 + \varepsilon)$-approx in subquadratic time (Cheng, Huang & Zhang, 2025)





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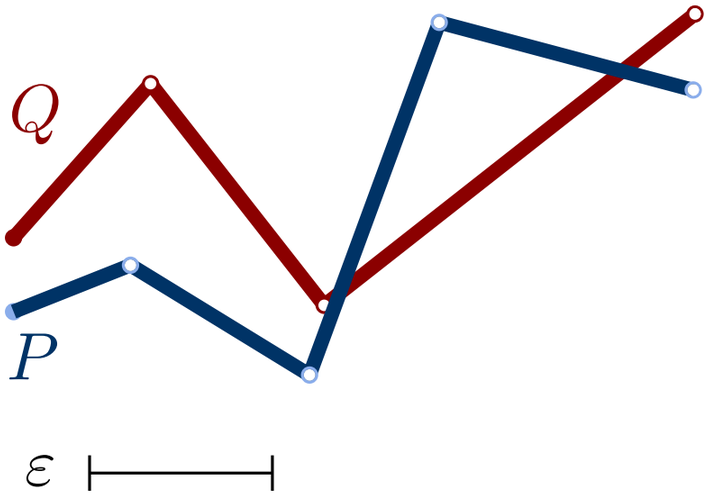
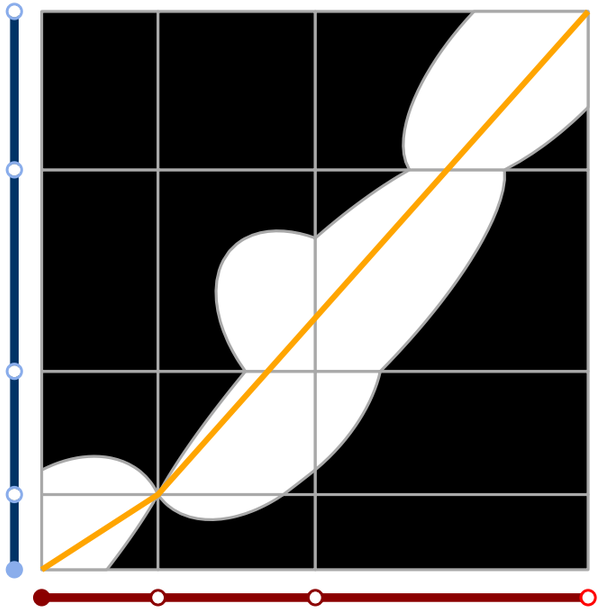


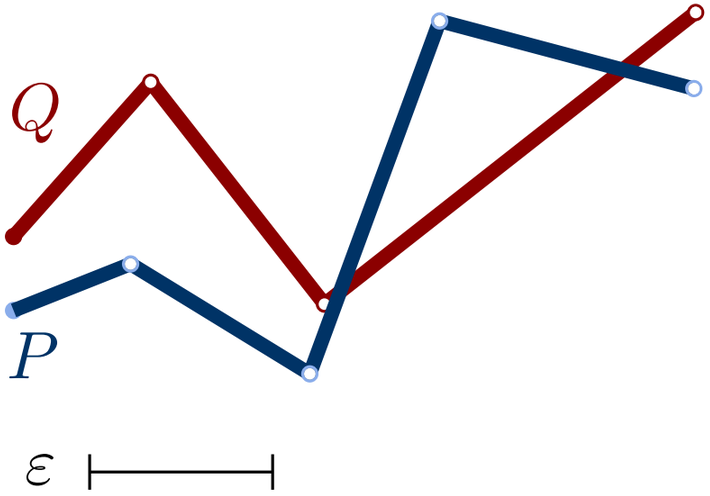


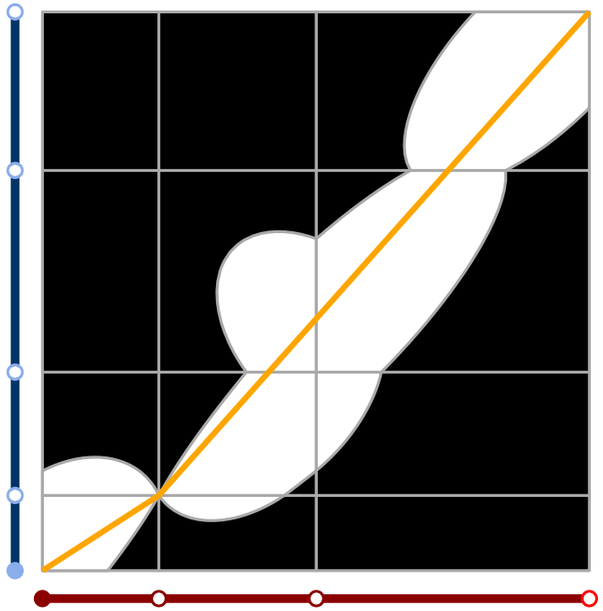
combines

- simplification
- grids
- greedy (for long edges)

Implementation

How to efficiently implement the algorithms?





Implementation

RUB

How to efficiently implement the algorithms?

ACM SIGSPATIAL Cup 2017

Home | Results | Problem Definition | Downloads | Submission and Evaluation | Organizing Committee

Trajectory similarity is a very basic problem when dealing with this type of data and many different ways of comparing trajectories have been proposed. In this challenge, we concentrate on a very intuitive measure called **Fréchet distance** with hard computational complexity, but superior quality in theory and practice.

Paper of the Winning Teams

All winning teams were invited to publish a paper describing their approach in the proceedings of SIGSPATIAL GIS.

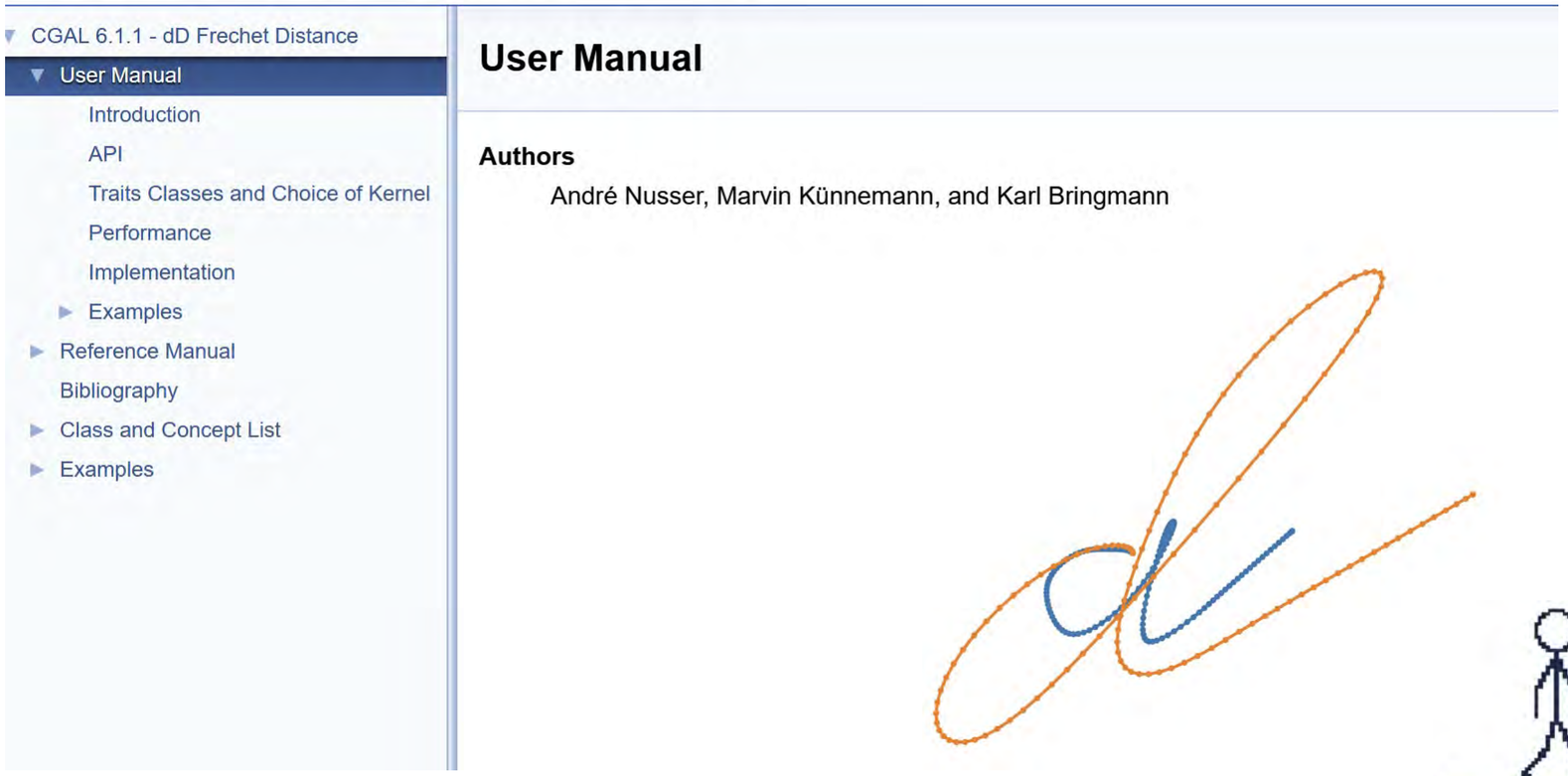
- **A fast implementation of near neighbors queries for fréchet distance.**
  J. Baldus and K. Bringmann.
- **Efficient trajectory queries under the fréchet distance.**
  K. Buchin, Y. Diez, T. van Diggelen, and W. Meulemans.
- **A filter-and-refinement-algorithm for range queries based on the fréchet distance.**
  F. Dütsch and J. Vahrenhold.

Implementation

How to efficiently implement the algorithms?

- Walking the Dog Fast in Practice: Algorithm Engineering of the Fréchet Distance (Bringmann, Künnemann, Nusser, 2019)

CGAL 6.1.1 - dD Frechet Distance



Computation Algorithm

RUB

Input: Polygonal curves P, Q
Output: $d_F(P, Q)$
determine all critical values
sort the critical values
do a **binary search** over the values,
solving the decision problem in each step

Runtime: $O(n^3 \log n)$

faster: with *parametric search* in $O(n^2 \log n)$ time

Theorem (Alt, Godau '95)
For two polygonal curves P, Q of size n, one can

- decide $d_F(P, Q) < \varepsilon$ in $O(n^2)$ time
- compute $d_F(P, Q)$ in $O(n^2 \log n)$ time

Questions:

- faster!
- implementation!
- generalizations?
- applications?

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Questions:

- faster!
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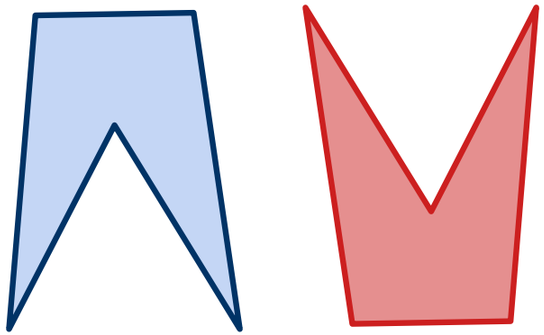
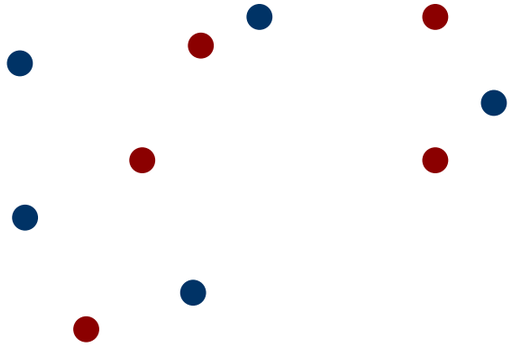
- **applications?**

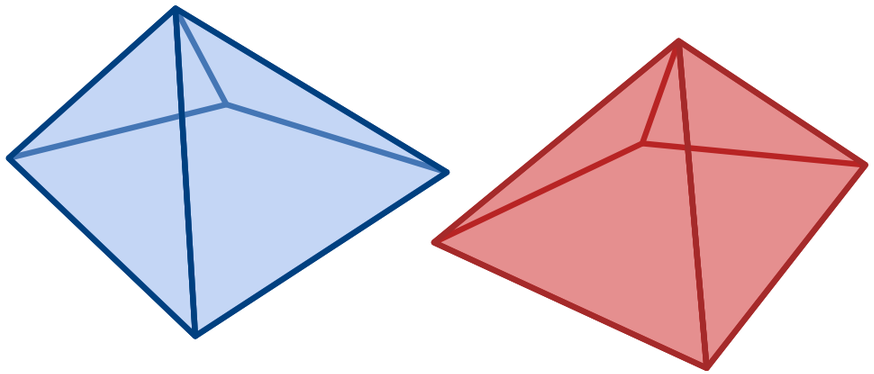
Generalizations

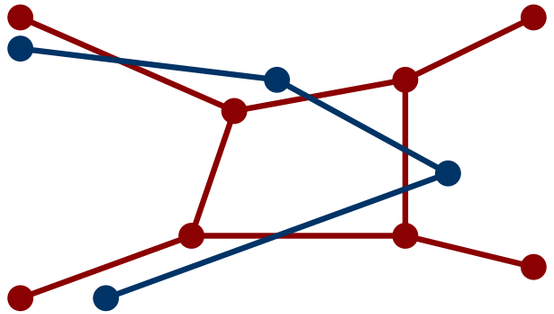
RUB

What about other shapes, e.g., surfaces, polygons, graphs, or point sets?





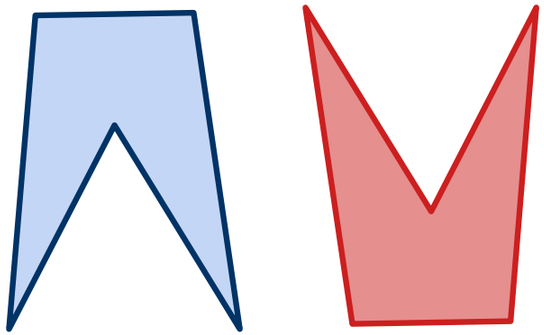


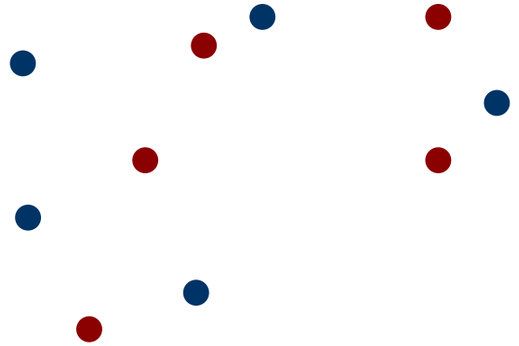


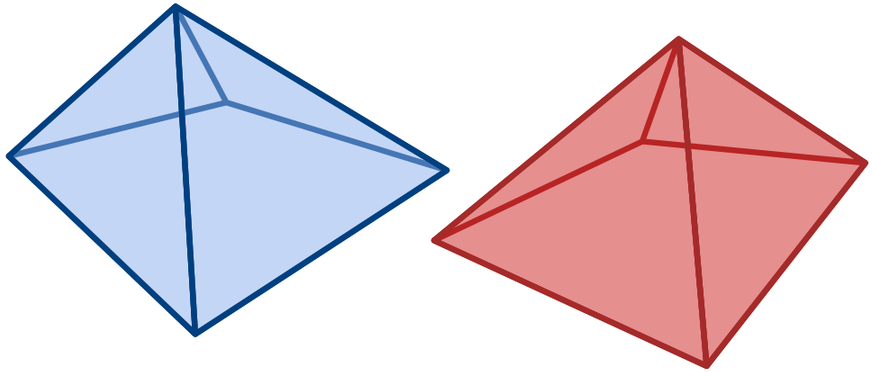
Generalizations

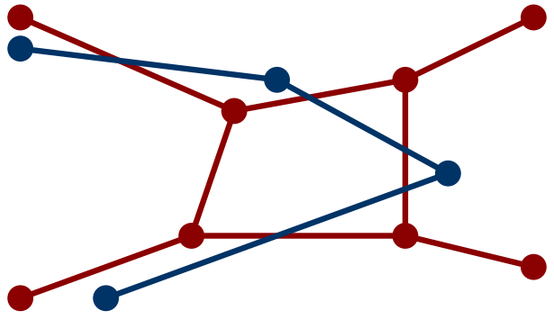
What about other shapes, e.g., surfaces, polygons, graphs, or point sets?

$\rightarrow$ Fréchet distance has been generalized to all of these









several of these turn out to be NP-hard

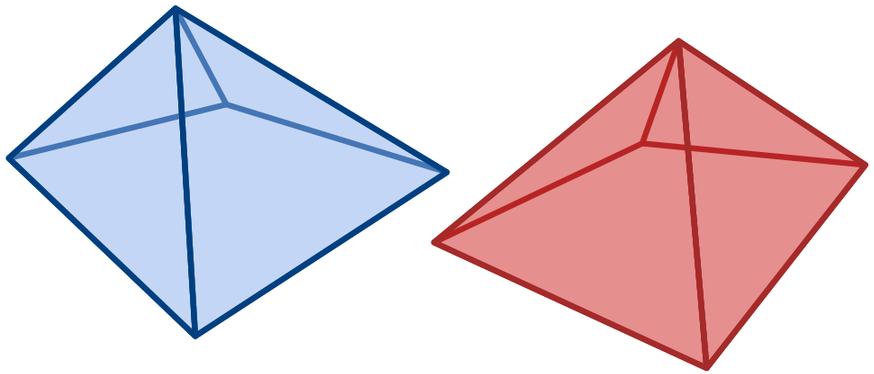
Generalizations

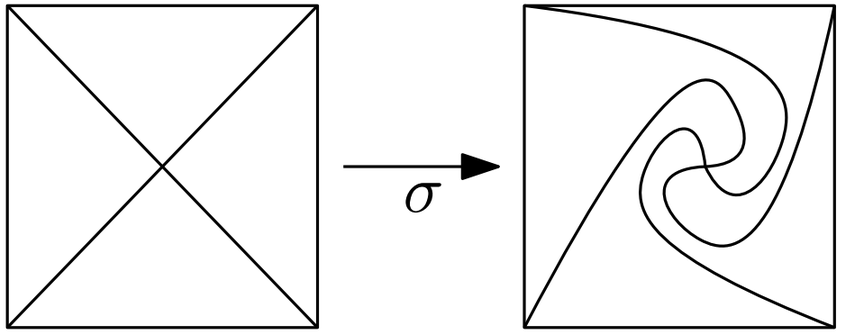
RUB

What about other shapes, e.g., **surfaces**, polygons, graphs, or point sets?

$\rightarrow$ Fréchet distance has been generalized to all of these

- generalized to surfaces (Fréchet, 1924)
- NP-hard (Godau, 1998), (Buchin, Buchin, Schulz 2010)
- Semi-computable (Alt, Buchin, 2004)
- Computable (Nayyeri, Xu, 2016)



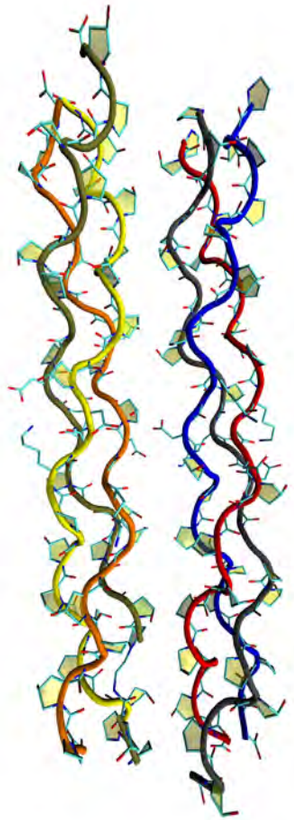


Homeomorphisms on $[0,1]^2$ are not as nice as on $[0,1]$

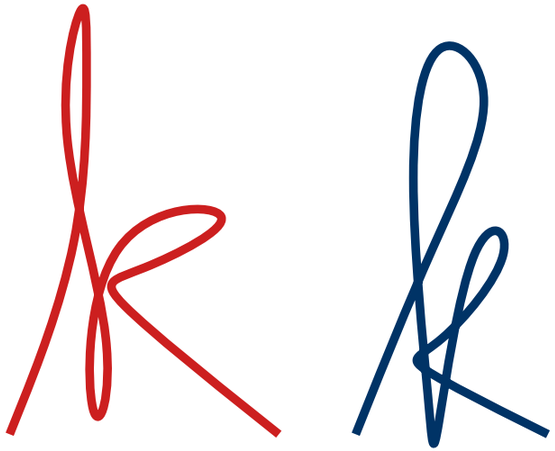
Under Transformation

RUB

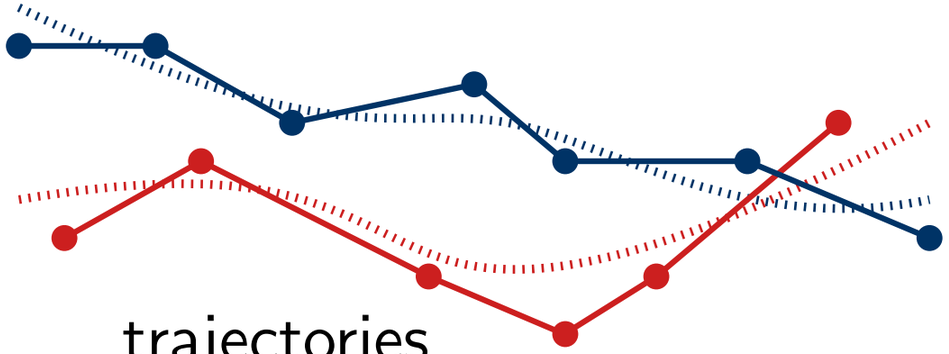
Shape matching: given two curves, find a transformation minimizing their distance.



protein backbones



handwriting



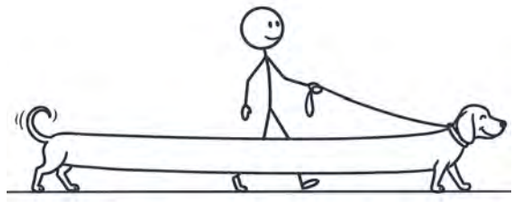
trajectories

Under Transformation

RUB

Fréchet distance of Curves under Transformations

- in 2D in $\tilde{O}(n^8)$ time (Alt, Knauer, Wenk, 2001)
- in 1D in $\tilde{O}(n^{2+\frac{2}{3}})$ time (Blank et al., 2025)
- in 2D in $\tilde{O}(n^{7+\frac{1}{3}})$ time (Buchin et al., 2025)



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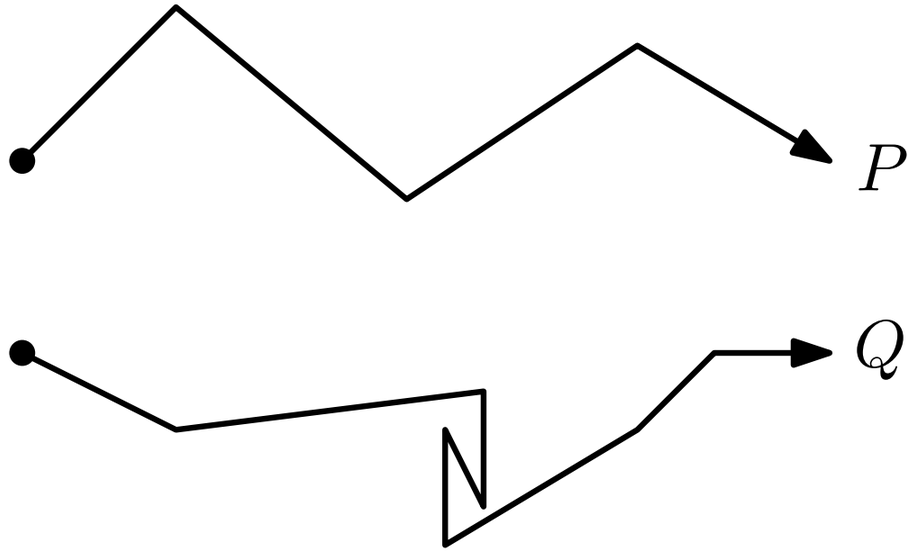

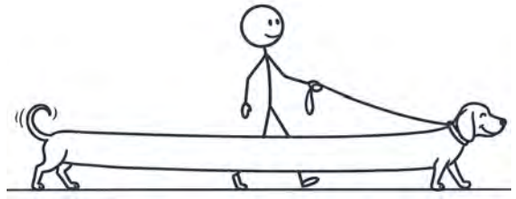

Under Transformation

RUB

Fréchet distance of Curves under Transformations

- in 2D in $\tilde{O}(n^8)$ time (Alt, Knauer, Wenk, 2001)
- in 1D in $\tilde{O}(n^{2+\frac{2}{3}})$ time (Blank et al., 2025)
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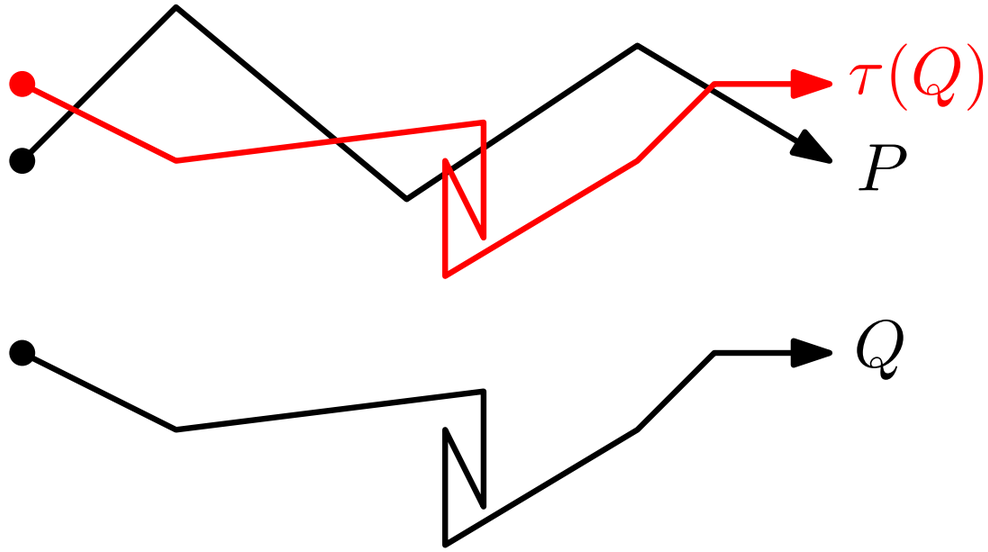

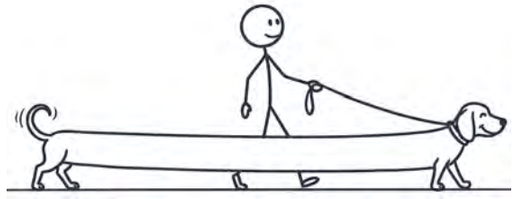

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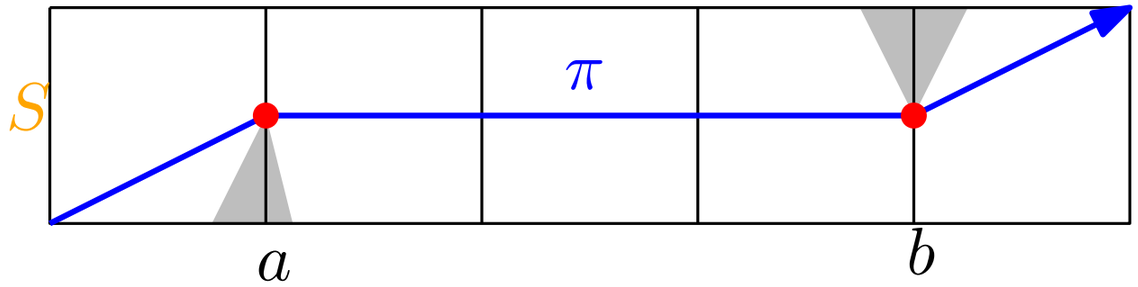
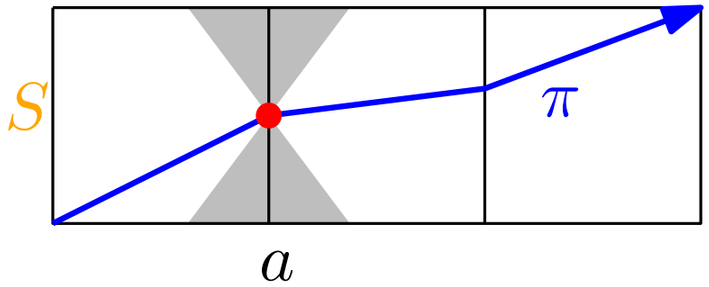
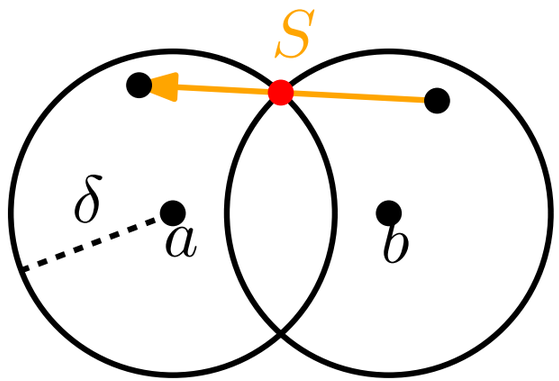
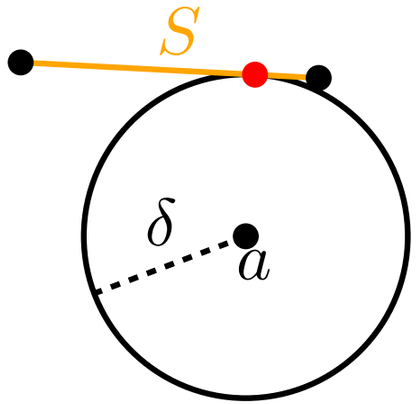
which translations change reachability in fsd?









Under Transformation

RUB

Fréchet distance of Curves under Transformations

- in 2D in $\tilde{O}(n^8)$ time (Alt, Knauer, Wenk, 2001)
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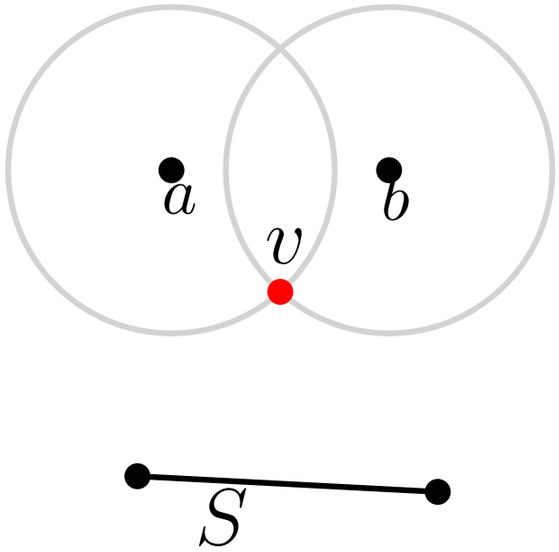

Geometric space

Translation space

Under Transformation

RUB

Fréchet distance of Curves under Transformations

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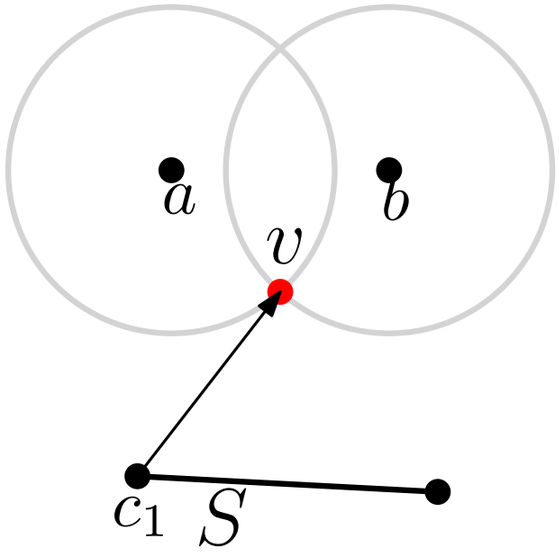
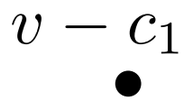


Geometric space



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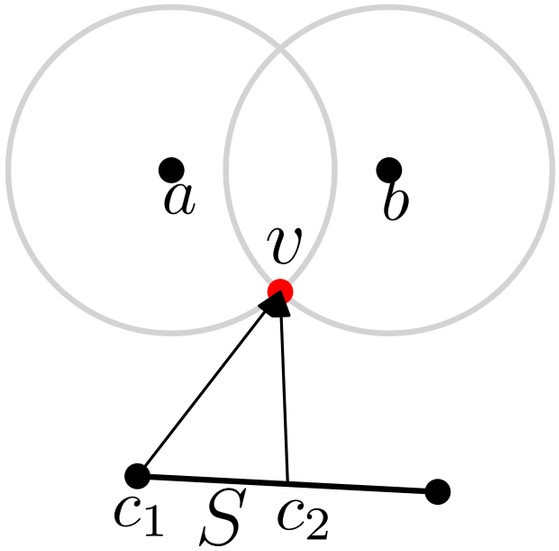
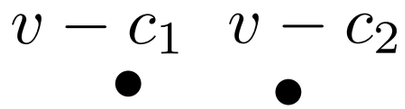


Geometric space



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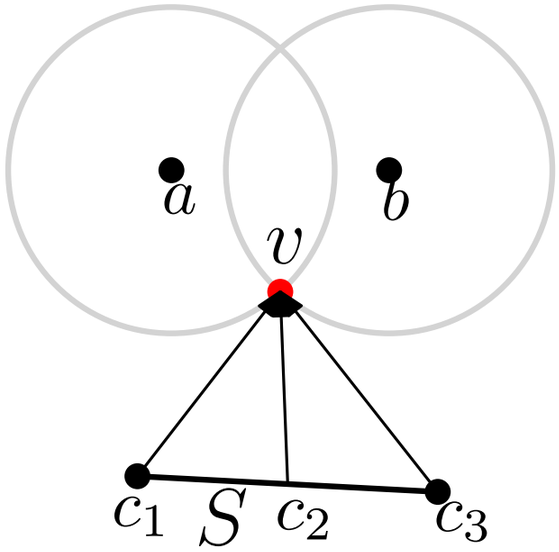
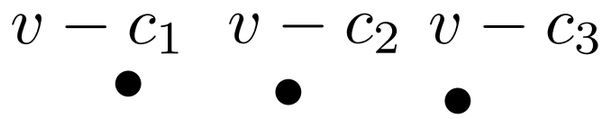


Geometric space



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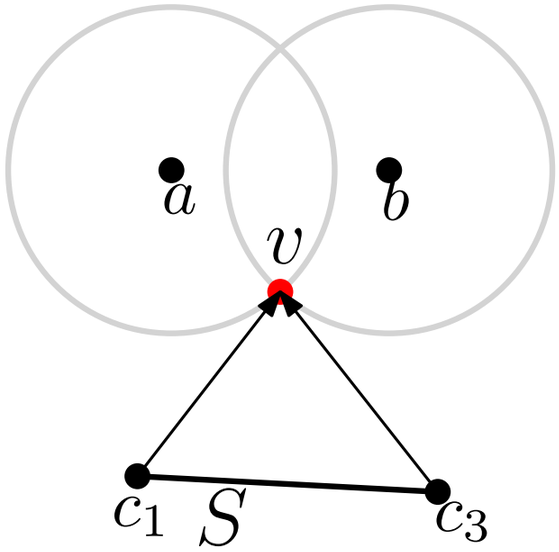

Geometric space

- $O(n^3)$ translations in total
- $O(n^2)$ pairs of vertices on P
- $O(n)$ segments on Q

Translation space

Under Transformation

RUB

Fréchet distance of Curves under Transformations

- in 2D in $\tilde{O}(n^8)$ time (Alt, Knauer, Wenk, 2001)
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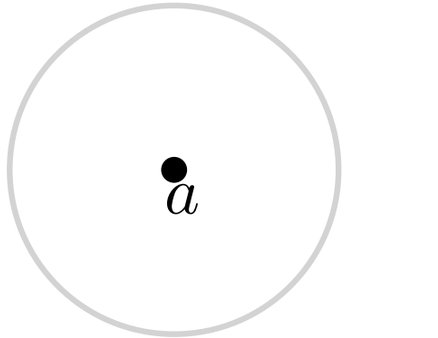

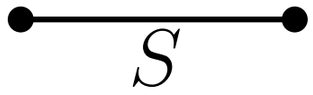

Geometric space

Translation space

Under Transformation

RUB

Fréchet distance of Curves under Transformations

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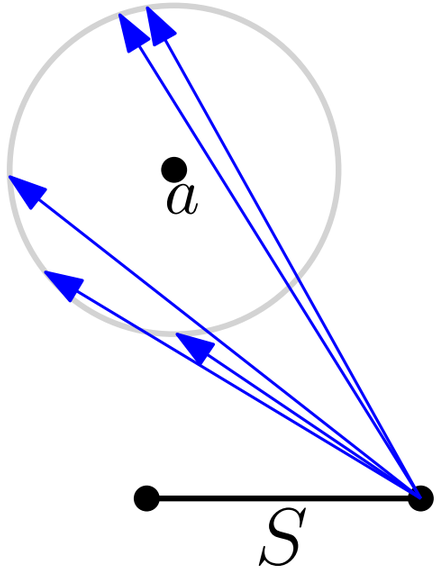
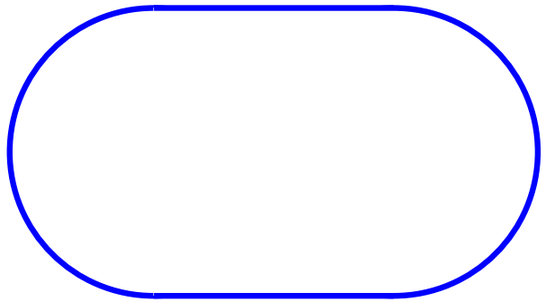


Geometric space



Translation space

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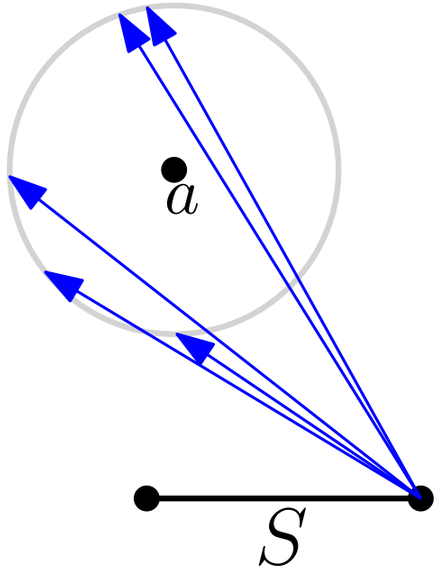
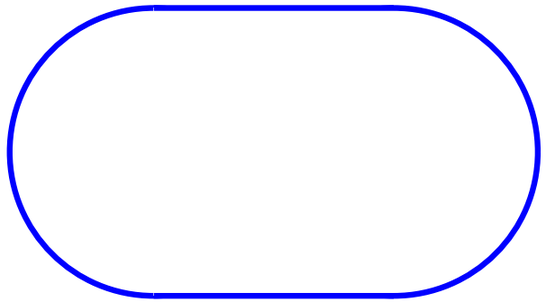


Geometric space

- $O(n^2)$ racetrack translations in total
- $O(n)$ vertices on Q
- $O(n)$ segments on P



Translation space

Under Transformation

RUB

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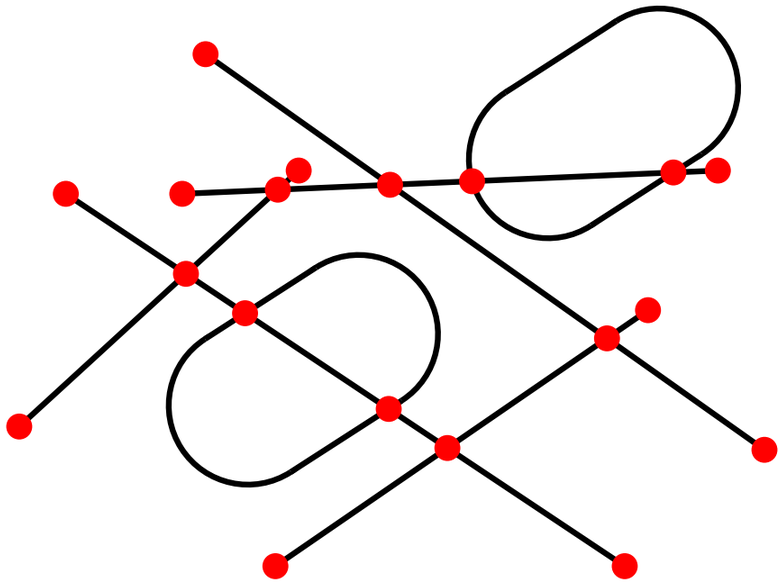

Translation space

Under Transformation

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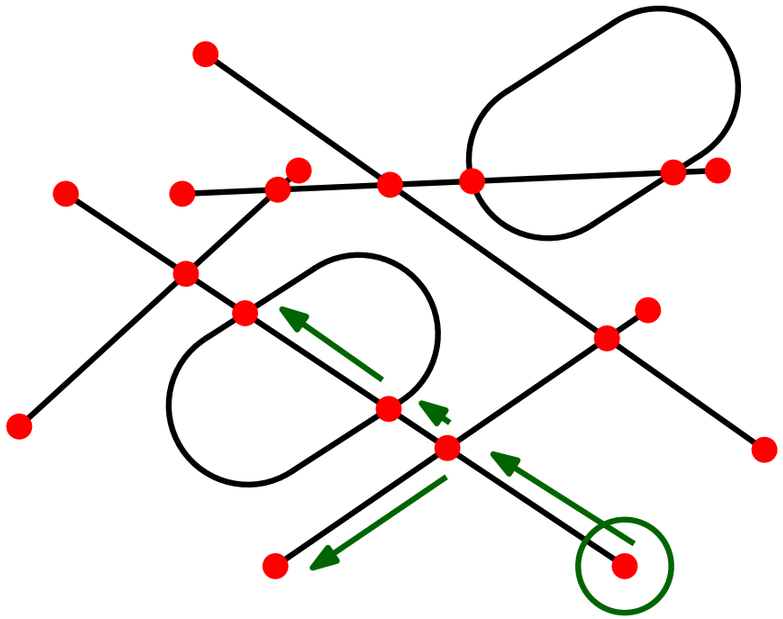

Translation space

1. Pick a starting intersection, and explore the arrangement of size $O(n^6)$ using DFS
2. For each intersection, update the freespace diagram and query reachability in $O(n^{\frac{4}{3}})$ time.

Under Transformation

RUB

Fréchet distance of Curves under Transformations

- in 2D in $\tilde{O}(n^8)$ time (Alt, Knauer, Wenk, 2001)
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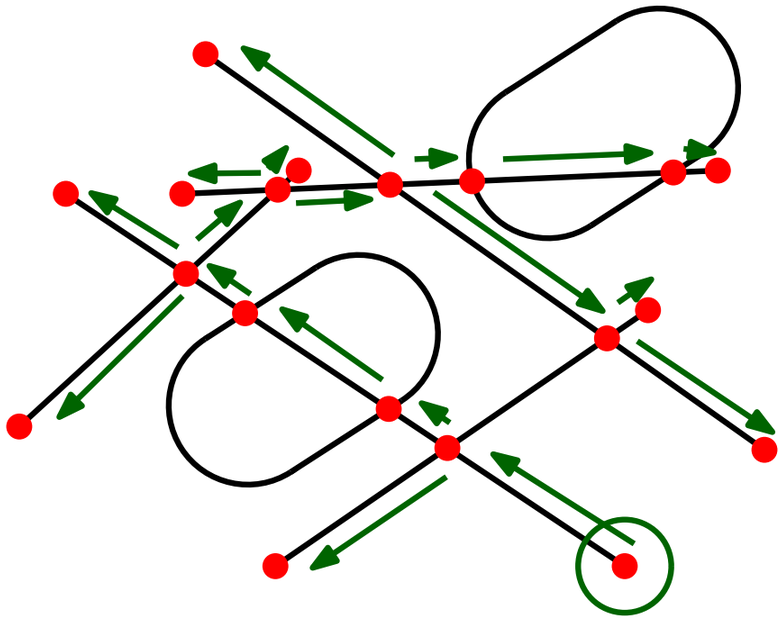
which translations change reachability in fsd?



Translation space

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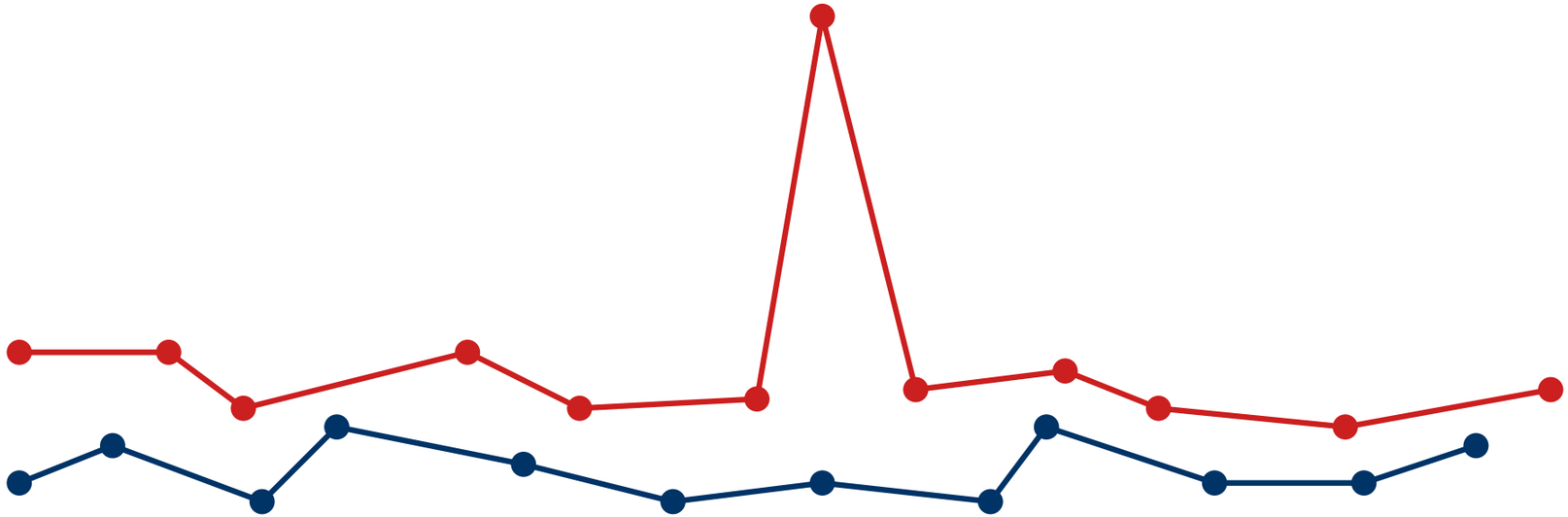
Less Sensitive to Outliers

RUB

Fréchet distance is a bottleneck measure



Less Sensitive to Outliers

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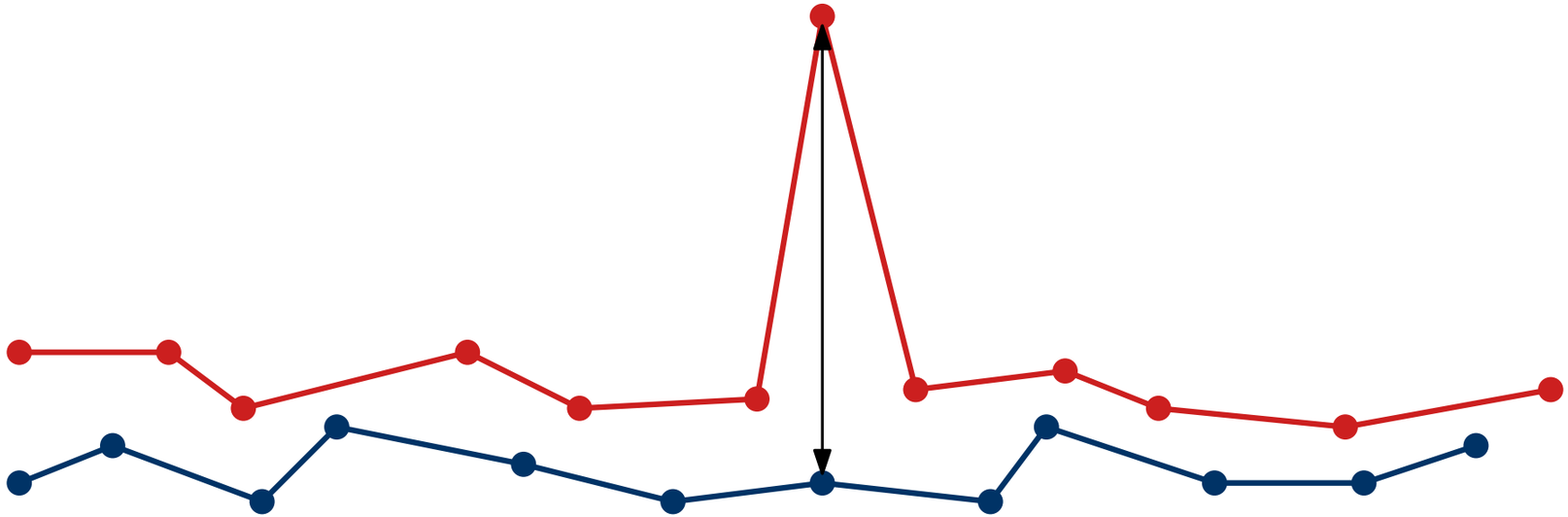
Fréchet distance is a bottleneck measure



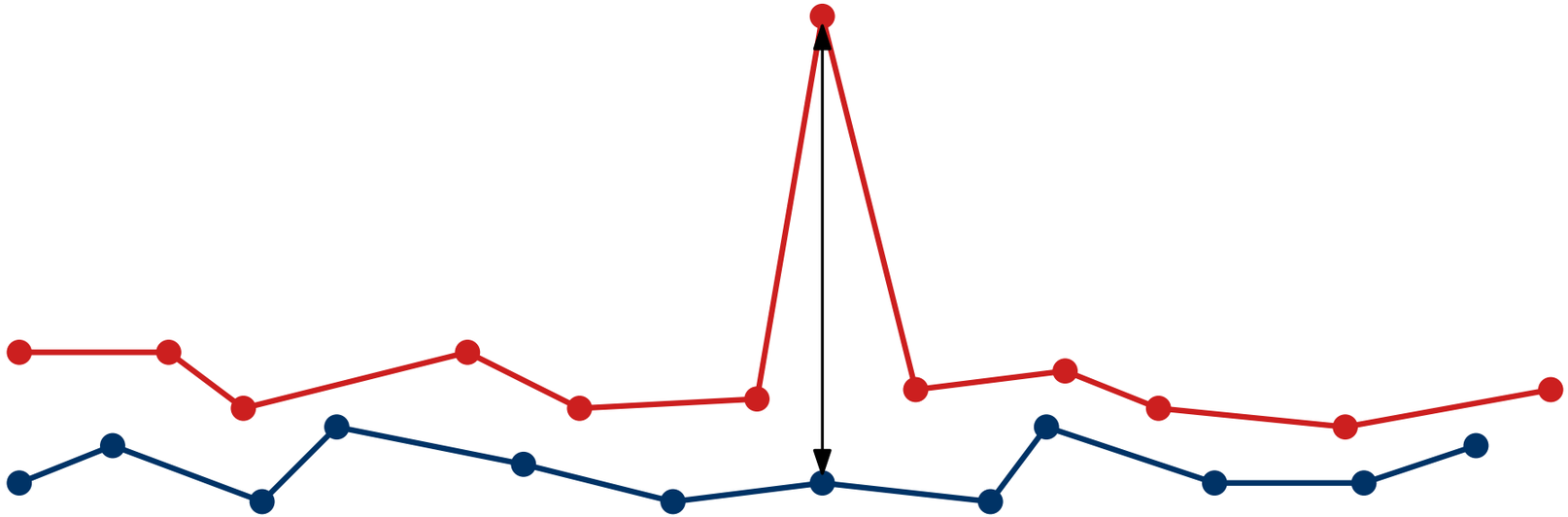
Less Sensitive to Outliers

RUB

Fréchet distance is a bottleneck measure

How to handle this?

- ignore parts of the curve
- take an average



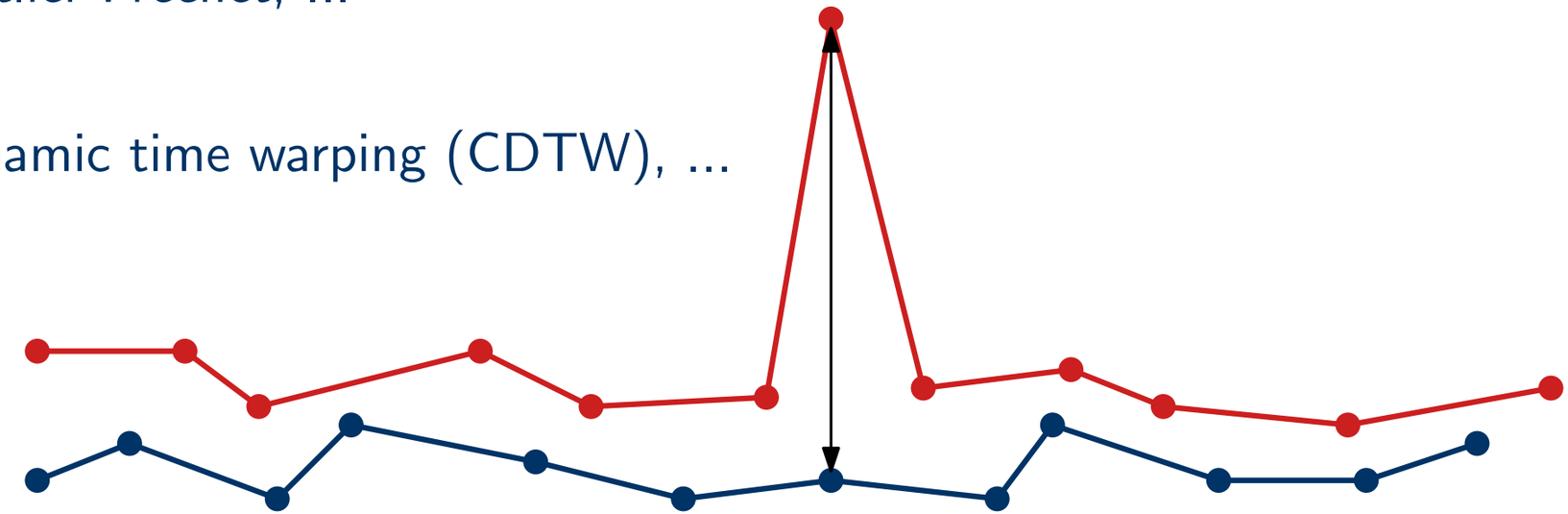
Less Sensitive to Outliers

RUB

Fréchet distance is a bottleneck measure

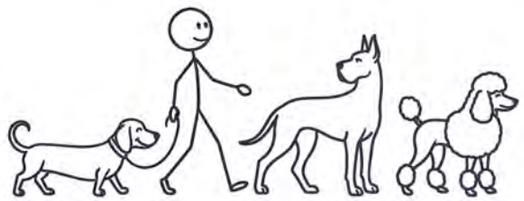
How to handle this?

- ignore parts of the curve
  partial, k-shortcut, k-outlier Fréchet, ...
- take an average
  average, continuous dynamic time warping (CDTW), ...



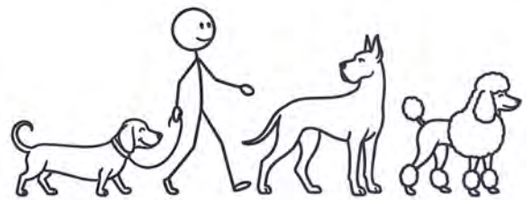
Variants

How can the Fréchet distance be adapted?



Variants

- weak [1992, Alt & Godau]



Variants

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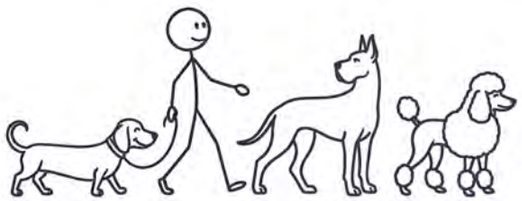

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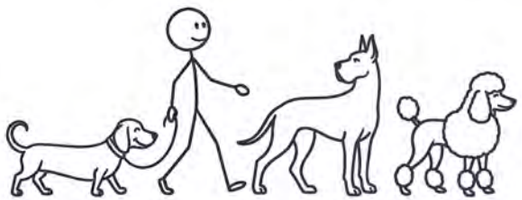

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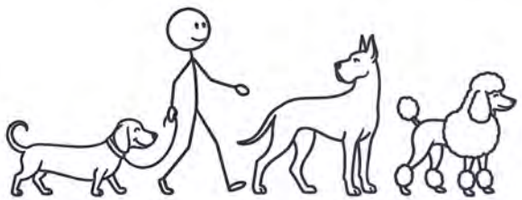

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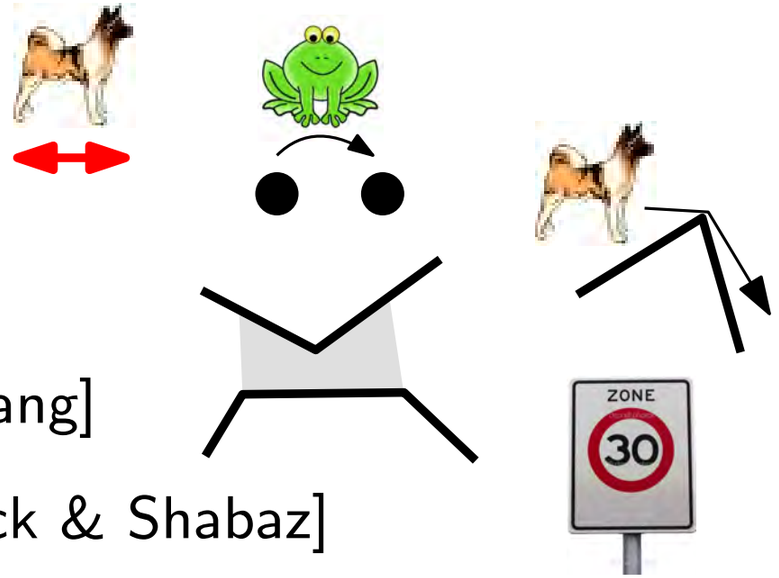

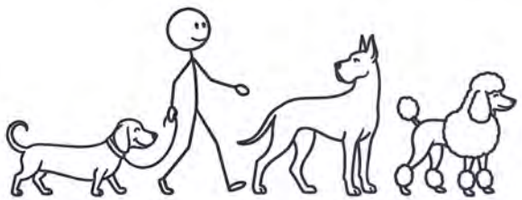

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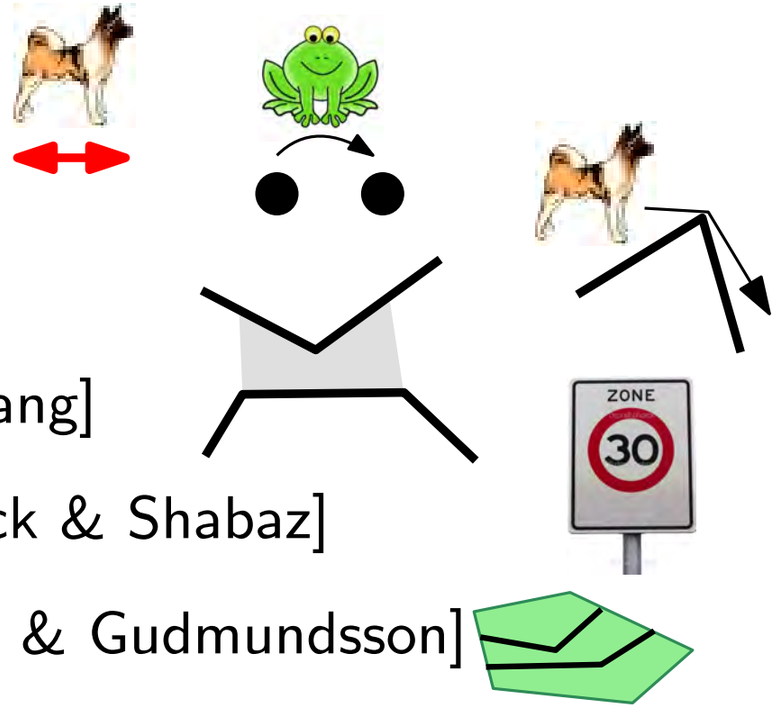

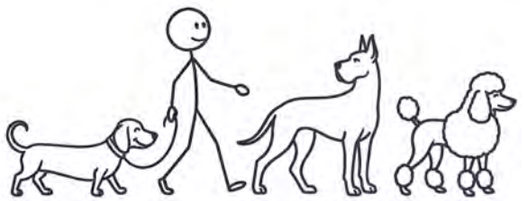

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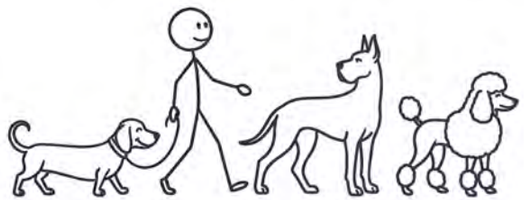

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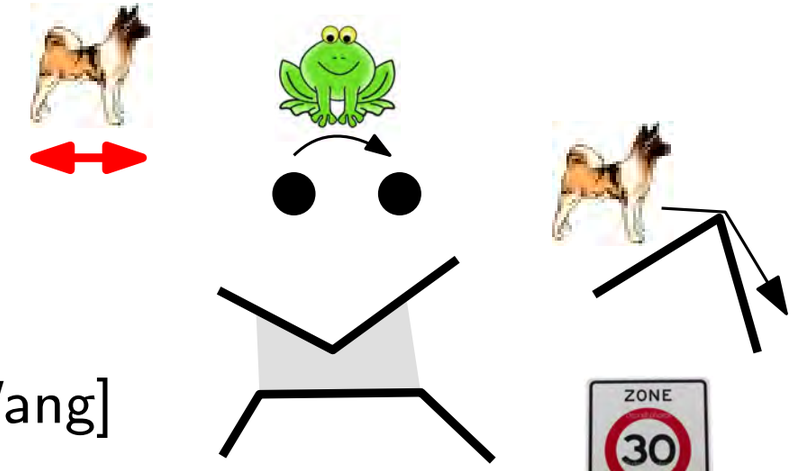
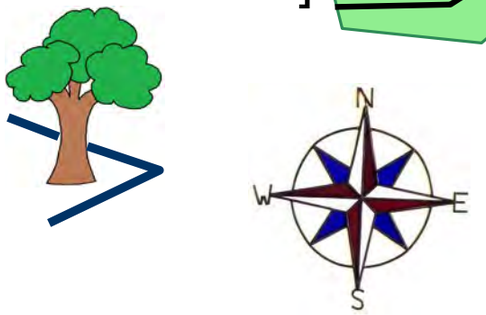
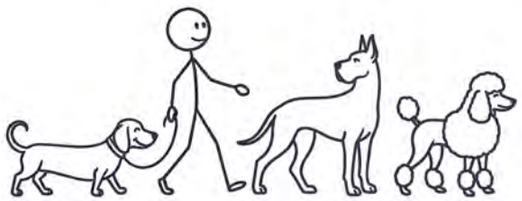
- direction-based [2011, de Berg & Cook]







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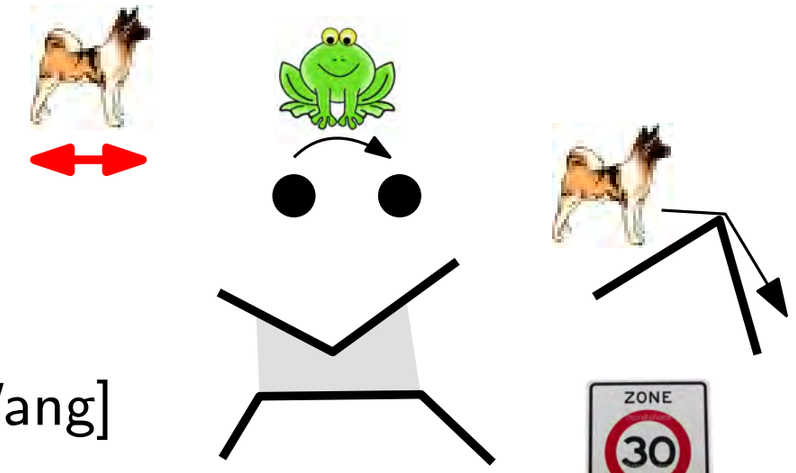

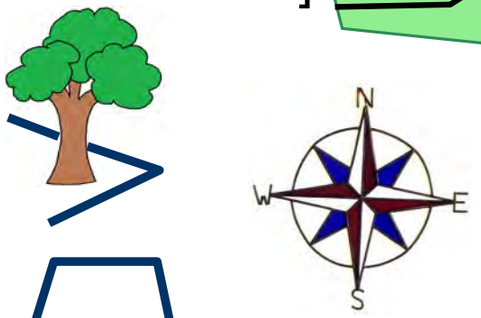

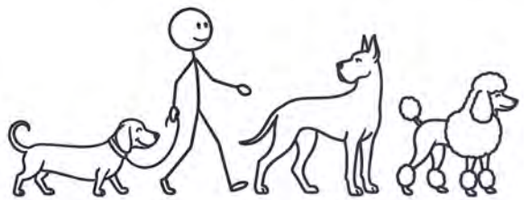

Variants

- weak [1992, Alt & Godau]
- discrete [1994, Eiter & Mannila]
- geodesic [2008, Cook & Wenk]
- partial [2009, Buchin, Buchin & Wang]
- speedlimits [2009, Maheshwari, Sack & Shabaz]
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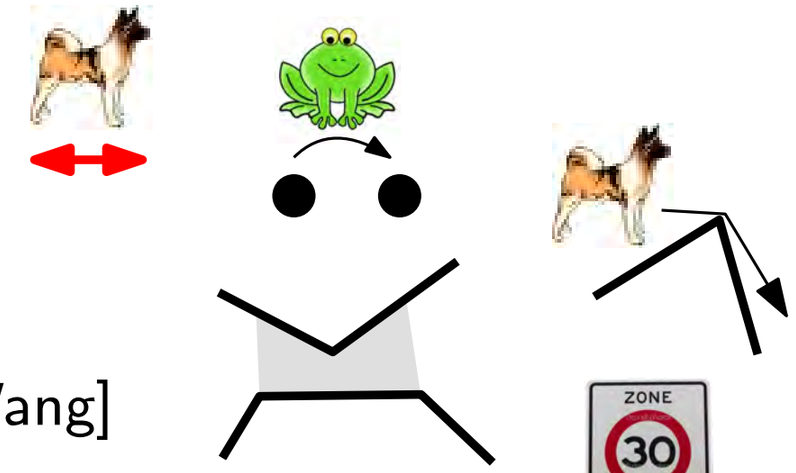
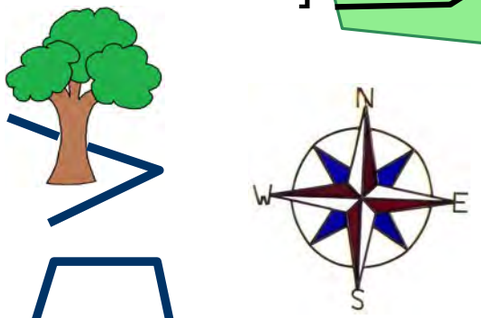
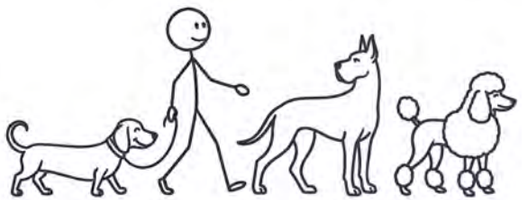
- shortcut [2012, Driemel & Har-Peled]
- ... 14 more years of variants ..

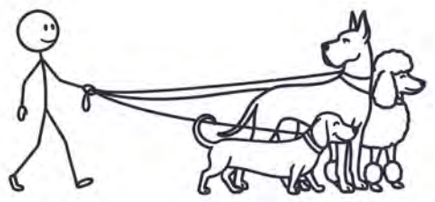






Tasks using similarity

Similarity used in many other tasks for example



Tasks using similarity

RUB

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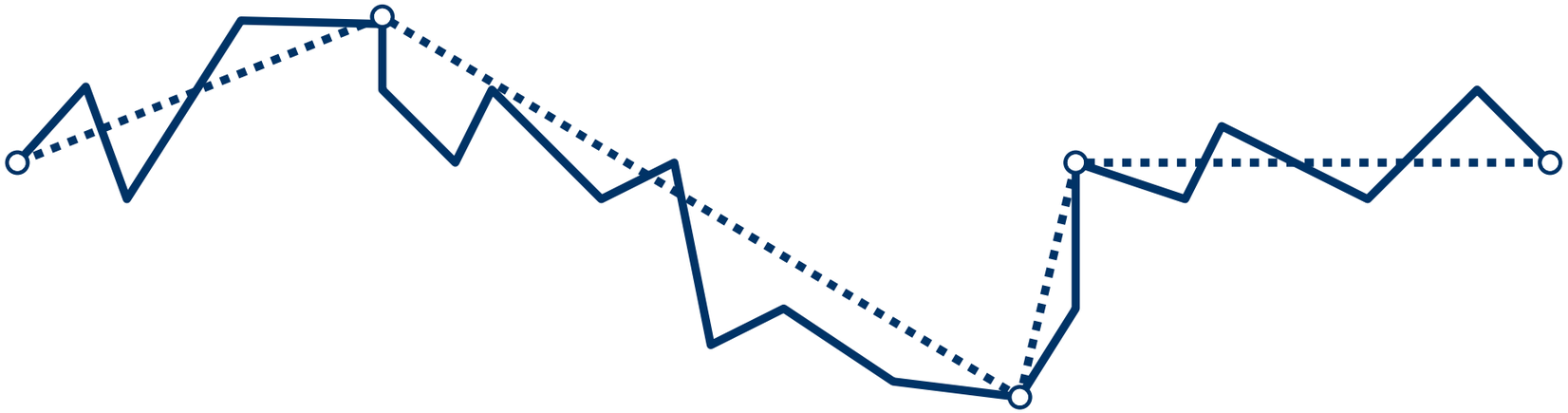
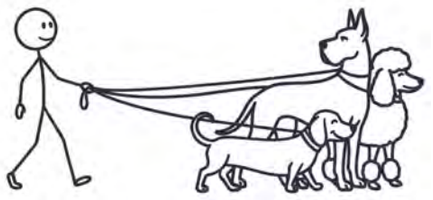
- **simplification:** reduce complexity with high similarity





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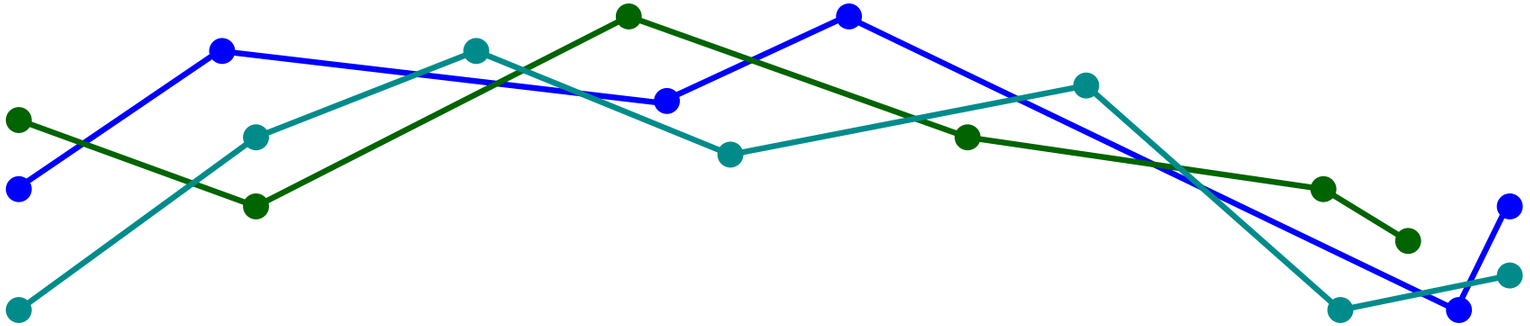

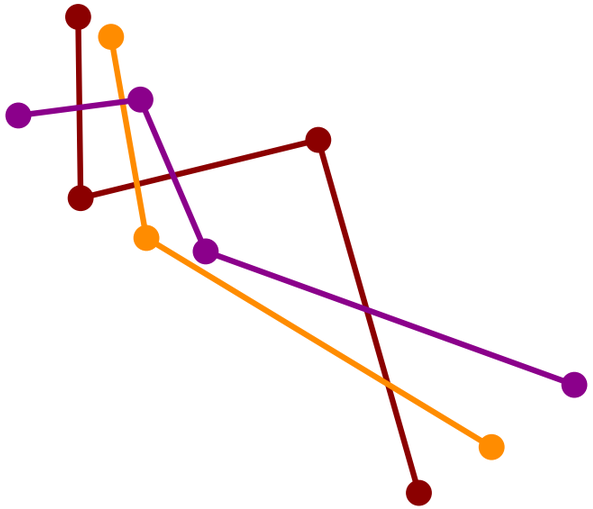

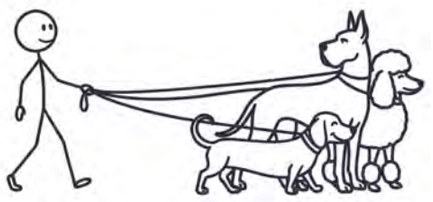

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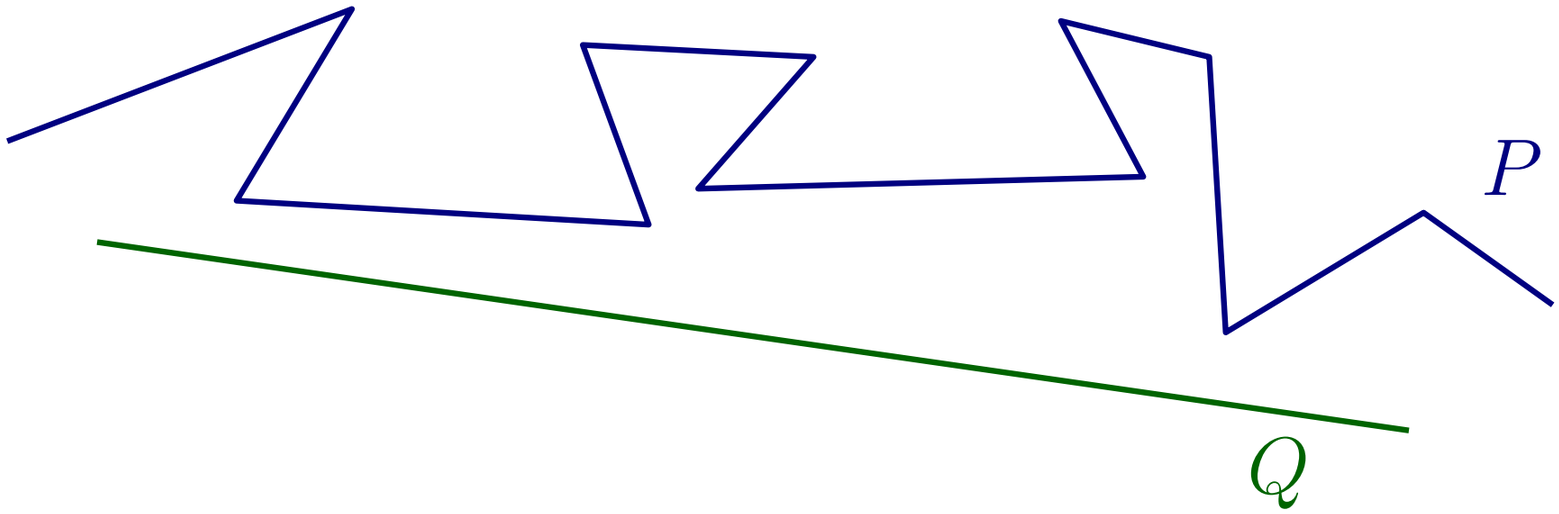

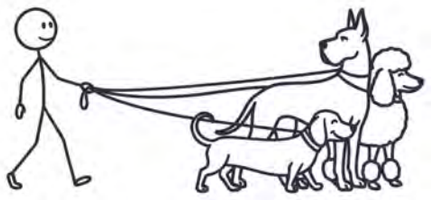

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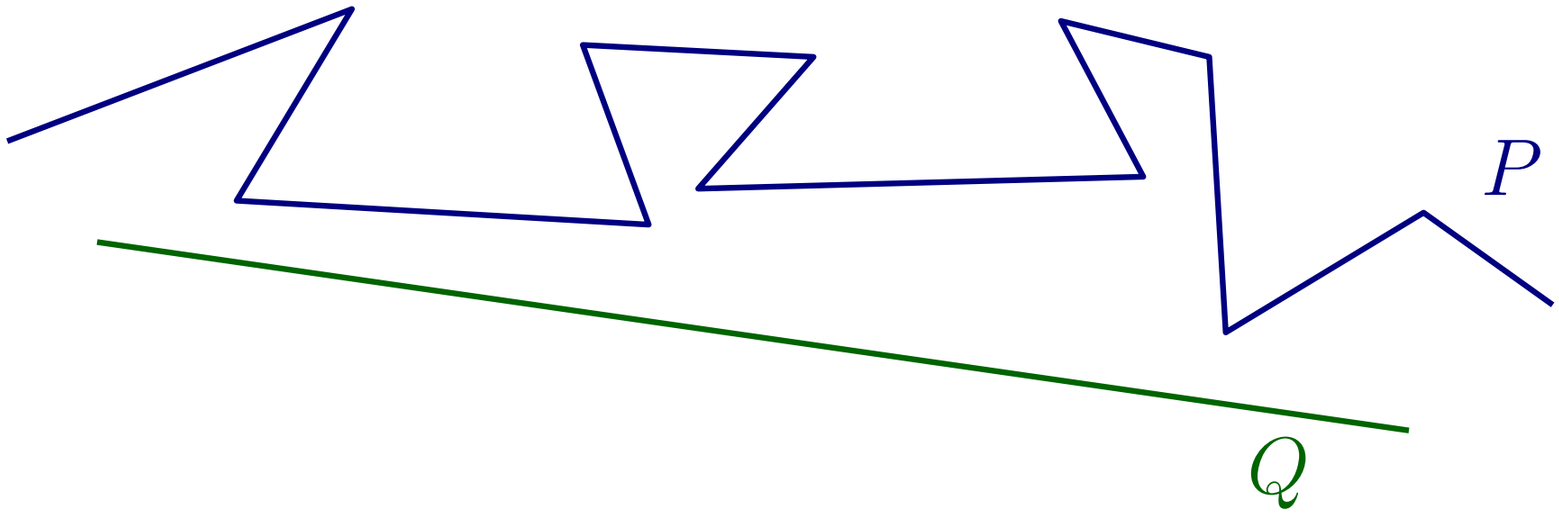
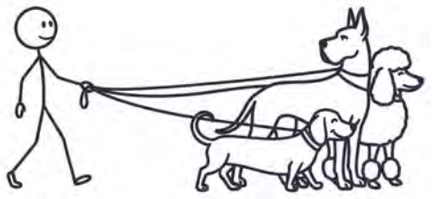


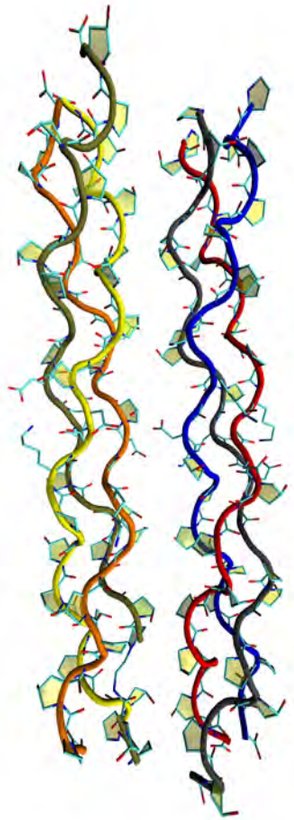
many interesting settings and results



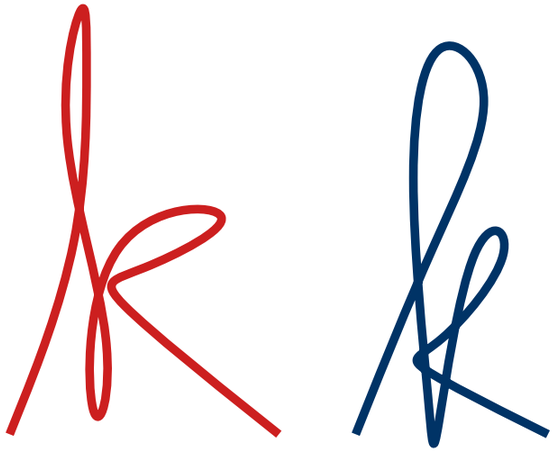
Applications

RUB

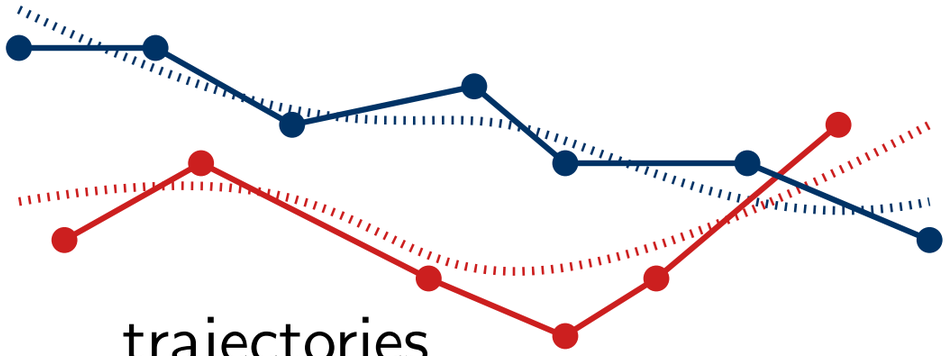
many applications, for instance..



protein backbones



handwriting



trajectories

Applications: Road Network Analysis

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many entities moving
on a network

Applications: Road Network Analysis

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Questions

- which path was taken?

Applications: Road Network Analysis

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- which path was taken?
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- how to compare two networks?

Applications: Road Network Analysis

RUB

many entities moving
on a network

Questions

- which path was taken? map matching
- which mode of transportation? segmentation, classification
- can we reconstruct the network? map construction
- how to compare two networks? map comparison

Summary & Open Problems

RUB

Fréchet distance has been well explored in the last 35 years!

much more research not mentioned here

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Thank you!

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