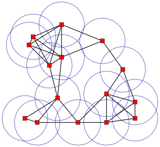
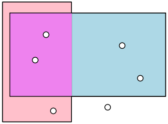
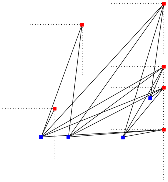
Compact Representations

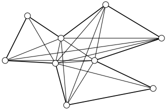
Jean Cardinal

Geometric relations

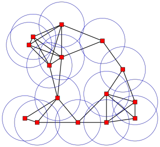

Intersection


Containment

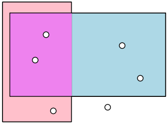

Dominance


Visibility

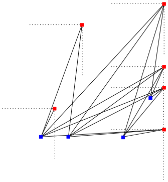
Geometric relations



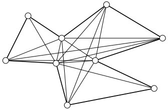
Intersection



Containment



Dominance



Visibility

- Possibly dense graphs
- Can we represent such graphs on n objects with $o(n^2)$ bits?
- Can we have practical such representations?

Geometric representations

Since our graphs are defined from n geometric objects, why not just store the objects?

Geometric representations

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Theorem

Unit disk and segment intersection representations require doubly exponential coordinates, hence exponentially many bits.

Kratochvíl and Matoušek, 1994 / McDiarmid and Müller, 2011

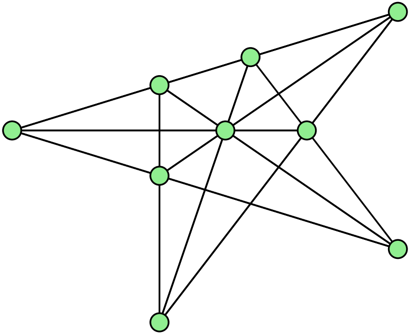
Theorem

Some polygon visibility graphs have geometric representations requiring exponential coordinates, hence linearly many bits per coordinate.

Lin and Skenia, 1995

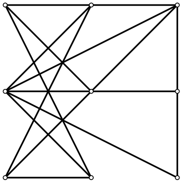
Irrationality

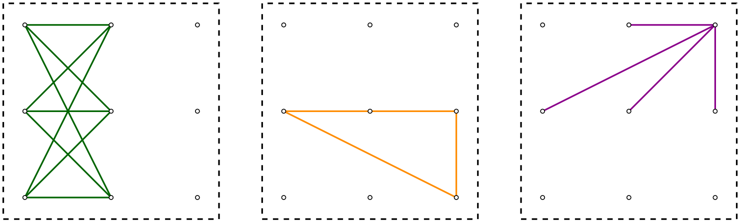
Every geometric representation of this point-line incidence graph requires an irrational coordinate.



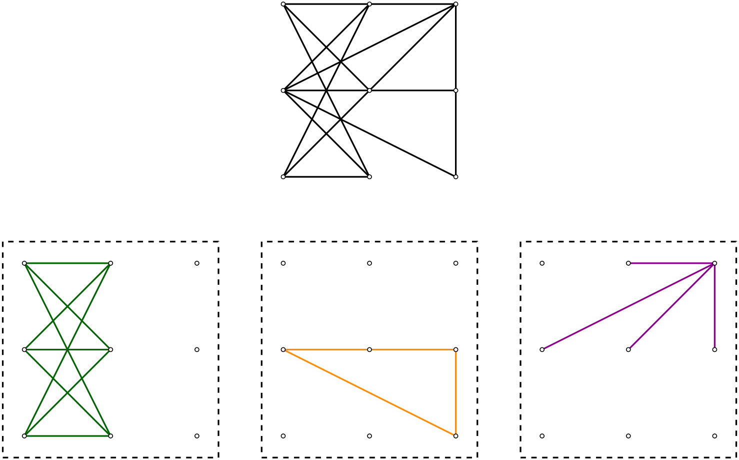
Perles, 1960s

Biclique covers





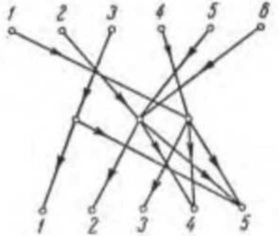
Biclique covers



size$= \sum_B |V(B)|$

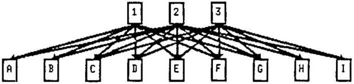
A not so new idea

$$\left\|\begin{matrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{matrix}\right\|$$

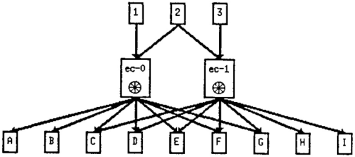


Lupanov, 1956

Graph drawing

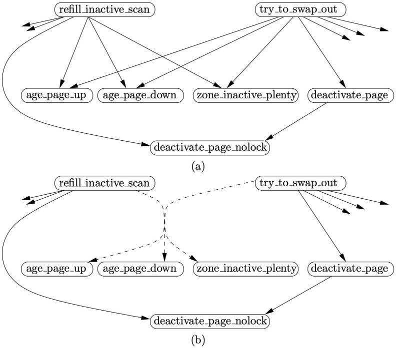


a (52 crossings)



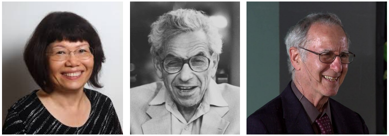
b (10 crossings)

Figure 3: A bipartite graph and the "covering by complete bipartite subgraphs" solution



Newberry, 1989 Dickerson, Eppstein, Goodrich, Meng, 2005

Known results on arbitrary graphs



Theorem

The worst-case size of a bipartite cover of a graph on n vertices is $\sim n^2/(2\log n)$

Chung, Erdős, Spencer, 1983

Krapivin, Przybocki, Sanhueza-Matamala, Subercaseaux, 2026

Theorem

Every graph G on n vertices has a biclique decomposition such that every vertex is contained in at most $O(n/\log n)$ bicliques.

Erdős and Pyber, 1997

Save space and time

Lemma

Given a biclique cover of size s of a graph G on n vertices, the breadth-first search tree rooted at any vertex of G can be computed in time $O(s)$, hence the (unweighted) all-pairs shortest paths problem can be solved in time $O(n \cdot s)$.

Feder and Motwani, 1995

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Feder and Motwani, 1995

Lemma

Given a biclique cover of size s of a bipartite graph G, the maximum matching problem on G can be solved in time $O(s^{1+\varepsilon})$ for any $\varepsilon > 0$.

Feder and Motwani, 1995 / Cabello, Cheng, Cheong, and Knauer, 2024

Zarankiewicz problem

Lemma

Let G be a graph without a $K_{t,t}$ subgraph, for some $t \in \mathbb{N}$, and with a biclique cover of size s. Then G has at most $t \cdot s$ edges.

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Proof.

For any biclique B in the cover, we have $|E(B)| \leq t \cdot |V(B)|$.
Hence, $|E(G)| \leq \sum_B |E(B)| \leq t \cdot \sum_B |V(B)| = t \cdot s$. □

Spanners

Lemma

If G has a biclique cover of size s then there is a subgraph G' of G which is a 3-hop spanner with s edges.

Conroy and Tóth, 2022

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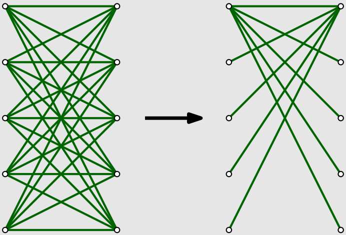

☐

Adjacency labeling

Associate a binary label to each vertex, in such a way that adjacency between two vertices can be deduced solely (and efficiently) from their labels.

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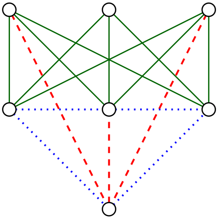
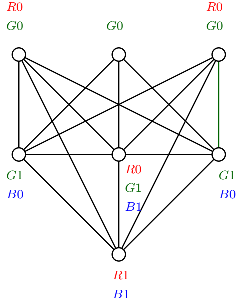
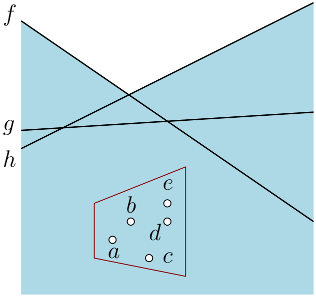
Lemma

If an n-vertex graph G has a biclique cover such that every vertex is contained in at most ν bicliques, then it has an adjacency labeling with labels of $O(\nu \cdot \log n)$ bits.





Bicliques from range searching



$\{a, b, c, d, e\} \times \{f, g, h\}$

Batch / offline range searching yields biclique encodings.

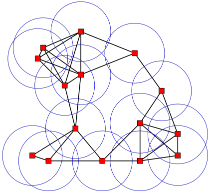
Agarwal, Ezra, Sharir, 2024

Semialgebraic graphs

- $V \subset \mathbb{R}^d$
- $uv \in E \Leftrightarrow$ Boolean formula in the signs of $O(1)$ polynomial functions of the corresponding pair of points.

Includes classes of intersection / containment / dominance graphs, but not visibility. Example:

$$xy \in E \Leftrightarrow (x_1 - x_2)^2 + (y_1 - y_2)^2 \leq 1.$$



Biclique covers of semialgebraic graphs

Theorem

Let G be a semialgebraic graph on n vertices of constant dimension d. Then for any $\varepsilon > 0$, G has a biclique cover of size $O(n^{2-2/(d+1)+\varepsilon})$.

Do, 2019 / Agarwal, Aronov, Ezra, Katz, Sharir, 2025

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Do, 2019 / Agarwal, Aronov, Ezra, Katz, Sharir, 2025

Theorem

Semialgebraic families of graphs of constant dimension d have adjacency labeling schemes with labels of $O(n^{1-2/(d+1)+\varepsilon})$ bits.

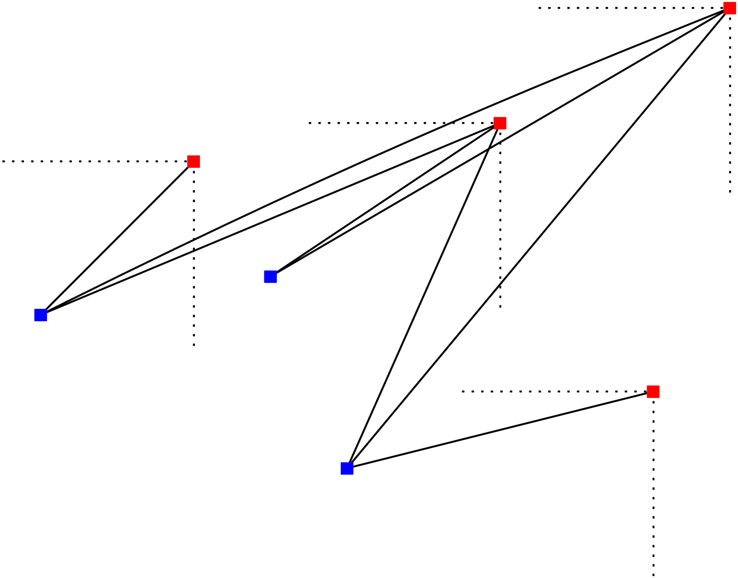
C. and Sharir, 2026

Via Polynomial partitioning.

Examples:

- Unit disk graphs and segment intersection graphs have adjacency labels of $O(n^{1/3+\varepsilon})$ bits.
- Disk graphs have adjacency labels of $O(n^{1/2+\varepsilon})$ bits.

Comparability bigraphs



Comparability bigraphs

Theorem

Any d*-dimensional comparability bigraph on* n *vertices has a biclique partition of size* $O(n \cdot \log^d n)$.

Comparability bigraphs

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Any d-dimensional comparability bigraph on n vertices has a biclique partition of size $O(n \cdot \log^d n)$.

Proof idea.

Divide-and-conquer + recurse on $d - 1$. ☐

Semilinear graphs

A *semilinear* graph family is a semialgebraic family for which all defining polynomials are linear.

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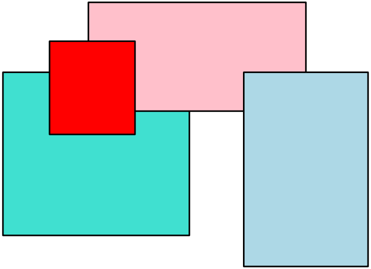
Interval graphs

Permutation graphs

Circle graphs

Box intersection graphs

Intersection graphs of rectilinear objects



Application to semilinear graphs

Theorem

Semilinear families have biclique covers of size $O(n \cdot \operatorname{polylog} n)$*, such that every vertex is contained in* $O(\operatorname{polylog} n)$ *bicliques.*

C. and Yuditsky, 2025

Application to semilinear graphs

Theorem

Semilinear families have biclique covers of size $O(n \cdot \operatorname{polylog} n)$, such that every vertex is contained in $O(\operatorname{polylog} n)$ bicliques.

C. and Yuditsky, 2025

Proof idea.

$f_i(x, y) \lesseqgtr 0$ can be rewritten $g_i(x) \lesseqgtr h_i(y)$, with g_i and h_i linear, and the Boolean formula can be rewritten in DNF. Hence we can decompose a semilinear graph into $O(1)$ comparability (bi)graphs. ☐

Zarankewicz problem for semilinear graphs

From our previous observation, we obtain a (simple) proof of the following.

Theorem

Let G be a semilinear graph on n vertices without a $K_{t,t}$ subgraph, for some $t \in \mathbb{N}$. Then G has at most $O(n \cdot \operatorname{polylog} n)$ edges.

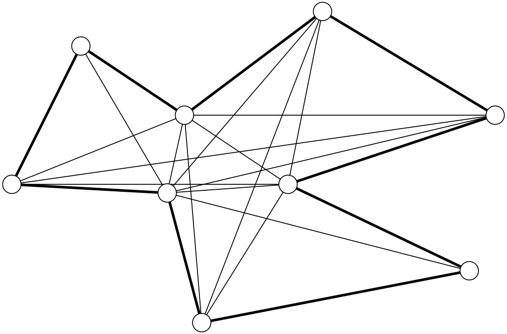
Basit, Chernikov, Starchenko, Tao, and Tran, 2021

Polygon visibility graphs

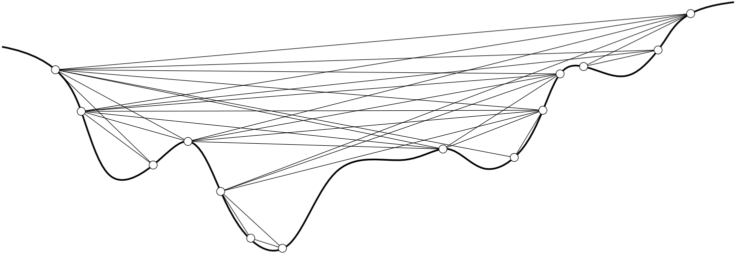
Theorem

Polygon visibility graphs have biclique covers of size $O(n \cdot \log^3 n)$.

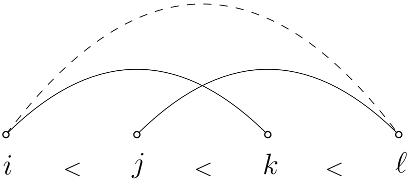
Agarwal, Alon, Aronov, and Suri, 1994



Capped graphs

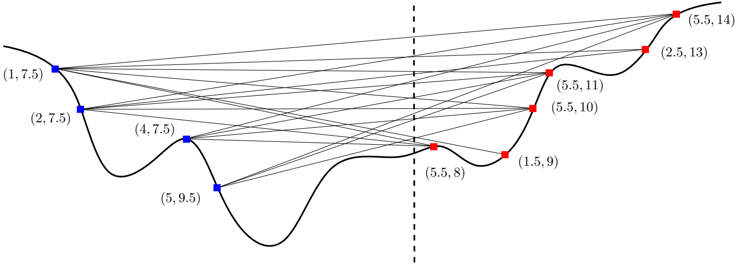


Forbidden ordered pattern:



Davies, Krawczyk, McCarty, Walczak, 2021

Capped graphs



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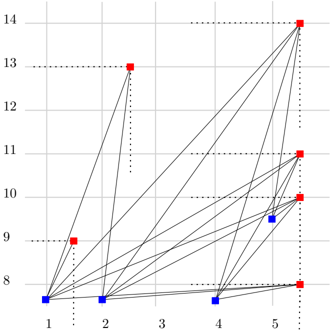

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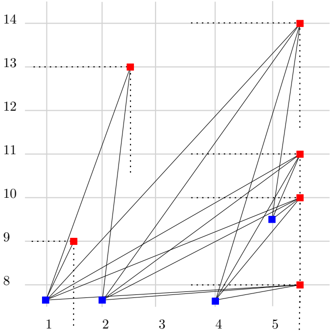

Theorem

Capped graphs have biclique covers of size $O(n \cdot \log^3 n)$*, such that every vertex is contained in* $O(\log^3 n)$ *bicliques.*

C. and Yuditsky, 2025

Lower bounds

Lemma

There exist unit disk graphs on n vertices, any biclique cover of which has size $\Omega(n^{4/3})$.

Erickson 1996 / C., Yuditsky, 2025

Lower bounds

Lemma

There exist unit disk graphs on n vertices, any biclique cover of which has size $\Omega(n^{4/3})$.

Erickson 1996 / C., Yuditsky, 2025

Proof idea.

From tight examples of the Szemerédi-Trotter incidence bound. ☐

Open problem

The speed of a family of graphs is the function $f(n)$ giving the number of graphs on n vertices in the class.

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Theorem

The speed of semialgebraic families of dimension d is

$$\Theta(2^{(1+o(1))dn\log n})$$

Sauermann, 2021

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The speed of semialgebraic families of dimension d is

$$\Theta(2^{(1+o(1))dn\log n})$$

Sauermann, 2021

Best lower bound on the size of adjacency labels: $\Omega(\log n)$.
Best upper bound: $O(n^{1-2/(d+1)+\varepsilon})$. Huge gap!

Thank You!

- Compact Representation of Semilinear and Terrain-Like Graphs, C. and Yuditsky, ESA'25.
- Implicit Representations via the Polynomial Method, C. and Sharir, Arxiv 2602.10922, 2026.
- Optimal and Efficient Partite Decompositions of Hypergraphs, Krapivin, Przybocki, Sanhueza-Matamala, and Subercaseaux, STOC'26.